HOW ELEMENTARY SCHOOL TEACHERS
LEARN TO TEACH MATHEMATICS

A DISSERTATION

submitted by

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Abstract

The education of elementary teachers in mathematics has emerged as a central issue in teacher education. Problems arise when teachers are asked to teach mathematics in a manner that emphasizes conceptual understanding and problem solving when their own mathematics education was procedurally based. Although there have been several studies that focus on the exposure of teachers to in-depth in-service programs, there has been less analysis of how most teachers develop their mathematical understanding.

This study examines how elementary school teachers learned to teach mathematics during their pre-service education and in their first few years of teaching. The study identifies those experiences teachers found to be most significant in their development as mathematics teachers. All seven teachers in this study had been taught using procedural methods and each had to find a way to integrate conceptual mathematics education into their own understanding. The study examines the teachers' motivations for pursuing this understanding and the circumstances that provided opportunities to do so. A qualitative analysis of these experiences, relying primarily on the teachers' own statements, reveals common features that allowed teachers to deepen their understanding of elementary mathematics.

Specifically, the study examined teachers during their student teaching and in their own classrooms two to five years later. The teachers were also interviewed twice about their experiences in learning and teaching mathematics. The results are presented as case narratives, primarily in the teachers' own words, as they reflected upon those experiences that critically affected their learning to teach mathematics. A cross-case analysis identified common themes:

1. All seven teachers had been very good mathematics students, yet six felt their own study of mathematics had not prepared them to teach conceptually. The teachers were motivated, as adults, to better understand elementary mathematics.

2. In their efforts to deepen their conceptual understanding the teachers relied on other teachers, the stimulus of challenging curricula, and pre-service courses and in-service activities emphasizing content.

3. In all cases the teachers deemed an adult re-thinking of elementary mathematics essential to develop successful pedagogical strategies.

The results raise questions for pre-service educators, principals, mentor teachers and curriculum developers.

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Chapter I Introduction

The purpose of this study is to provide new insights into how elementary school teachers develop their mathematical understanding and capacity to teach mathematics. The study is organized as a descriptive account of the experiences of seven Massachusetts elementary teachers who entered teaching in the mid 1990s. The analysis of the data collected reveals which experiences were significant in the teachers' development as teachers of mathematics. The source of this information is primarily the teachers themselves, reflecting on their own backgrounds and experiences. It is their stories I wish to tell, principally in their own words. It is my belief that reporting on what experiences teachers deem as having been the most instructive will provide useful information to those people who design and implement teacher preparation and professional development programs.

My research is presented as a series of case study narratives based on observations and interviews with these teachers. The narratives are based on recent interviews and telephone conversations as well as written records (from three to six years ago) compiled when these teachers were in a teacher preparation program in which I was their supervisor. I capture the teachers' voices as they relate

what they found to be significant experiences in their own elementary, high school, college, and teacher preparation education as well as in their more recent experience as practicing teachers. I include these narratives, which I present as roughly chronological biographies, to allow the reader to see each teacher as a whole.

Using cross-case analysis, I have extracted common themes in these teachers' experiences; ones that teachers found important for their development as teachers of mathematics. In the concluding chapter I present some implications of my findings in relation to strengthening the development of mathematical understanding and pedagogy in mathematics for elementary school teachers.

My interest in this topic came from two directions: one reflecting my concern with national policy issues and one reflecting more personal experiences as a teacher and teacher educator. There were major and pervasive changes in the 1990s in the United States regarding mathematics education for school children and these placed new demands on elementary teachers to use mathematics curricula and teaching methods that were very different from those used when they were students. The publication of the 1989 National Council of Teachers of Mathematics (NCTM) publication, *Curriculum and Evaluation Standards for School*

Mathematics, (NCTM, 1989) expressed a philosophy of mathematics education that moved away from rote procedural learning towards a philosophy of conceptual understanding. The new philosophy also included an appreciation of mathematics as problem solving, reasoning and communication, and stressed connections between different areas of mathematics and between mathematics and other disciplines. Today the debate over mathematics reform still rages. Teachers are often caught between their own experiences as learners of math and the different expectations embedded in new curricula. For this reason I was particularly interested in teachers who were caught in the middle of this mathematics reform movement, those who were elementary and secondary students before the reform and who became teachers after these changes were initiated. This particular group of teachers were taught elementary mathematics procedurally when they were elementary and secondary students; as teachers they were now asked to teach elementary mathematics for conceptual understanding. How did they combine their own mathematics education with the new demands of the national reform movement in mathematics education?

Teaching mathematics in a manner that fosters student understanding of mathematics requires a different kind of

knowledge than teaching mathematics by simply demonstrating algorithms and operations for students to memorize and apply. Proponents of various approaches to mathematics instruction for students seem to agree on the fact that teachers need to understand the mathematics they teach. However, researchers have repeatedly found that pre-service and practicing elementary teachers in the United States often lack conceptual understanding of the mathematics taught in elementary school. This calls into question whether teachers in fact have the kinds of underlying mathematics knowledge they need to teach mathematics for understanding (Ball, 1989; Ball, 1991; Ball & Wilson, 1990; Borko et al., 1992; Brown & Borko, 1992; CBMS, 2001a; Eisenhart et al., 1993; Mewborn, 2000; Simon, 1993). This background of concern and interest has formed the central research question for this study: How did these teachers contend with these issues? How did they learn the mathematics they needed and where did they learn it? Did the reforms inspire or demoralize the teachers? What stresses emerged from the disparity between how the teachers were taught and how they were expected to teach?

Additionally, there was a very personal impetus for this study. As a university supervisor of student teachers, as an educator of teachers in pre-service mathematics

methods classes, and as a high school mathematics teacher and an elementary school teacher for more than twenty years, I felt equipped to participate as an instructor of beginning teachers. In 1994, I began working as a university supervisor of student teachers who were training at both elementary and high school levels of certification. In 1995 and 1996, I also taught graduate classes in methods for teaching elementary school mathematics to both pre-service teachers (unlicensed prospective teachers taking courses to fulfill teacher certification requirements) and experienced elementary teachers in a masters degree program[1]. My classroom experience across grade levels gave me a good overview of the entire curricula and fostered an appreciation of the important role elementary school mathematics instruction plays both in laying a foundation for higher mathematics study and in influencing students' attitudes about learning mathematics.

I found the vast majority of my teacher education students were intelligent, competent, and confident in their abilities to teach most elementary subjects. Yet, despite these general qualities, many entered the mathematics methods classes with a dislike of mathematics. Many believed that they simply "could not do mathematics".

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[1] Math methods class students were not the same individuals I supervised as student teachers.

The curricula in all three schools of education for which I worked seemed to be based on the assumption that student teachers and teachers enter their classrooms with a full understanding of elementary mathematics subject matter. Yet, all of my experience suggested otherwise. I was surprised how little my own institution required in formal mathematics training on the college level. I carried out a survey of fifteen Massachusetts pre-service graduate elementary education programs, including the ones at which I worked. I found that these programs uniformly required prospective elementary teachers to take only a single course in how to teach mathematics and required no other mathematics courses in the program (unpublished data, 2000). However, contrary to what course requirements would imply, my experience with teachers matched the research findings documenting elementary teachers' limited conceptual understanding of mathematics. From my perspective as a supervisor, I was curious to know whether teachers found ways later as they were teaching to deepen and broaden their conceptual understandings of mathematics. I was particularly interested in learning what teachers thought about their mathematics knowledge and their preparation to teach mathematics.

I examined the views of teachers about the adequacy of their mathematics methods course (as a preparation for teaching elementary school mathematics) through two qualitative research studies in the fall of 1996. In a survey of eighteen pre-service teachers taking a math methods course and in an in-depth interview study of four veteran teachers who took a math methods course in late 1980s that stressed building mathematical understanding, I asked teachers to critique their math methods course as a vehicle for preparing them to teach mathematics.

Teachers in both groups viewed the math methods course as only a beginning in their learning to teach mathematics. With the exception of one student educated in England, the pre-service teachers said the course exposed them to a way of studying mathematics that was very different from methods and materials used when they were students. They reported that the pre-service course had in fact helped them begin to understand the mathematics underlying the computational procedures they had previously only memorized as students. Many of the pre-service teachers said if they were to redesign the course, they would design a two-semester methods course (unpublished data, 1996).

All four experienced teachers also said the methods course served more broadly as an introduction to thinking about mathematics and how to teach it, but they further stated that they learned mathematics and learned to teach mathematics mostly while student teaching and teaching. The experienced teachers also credited colleagues and support personnel, as well as various intensive in-service teacher professional development programs, as contributing greatly to their learning to teach mathematics (unpublished data, 1996).

What I heard consistently from the teachers in 1996 resonated with my own experience as a classroom teacher: that learning to teach mathematics is a life-long process affected by the classroom and school contexts (i.e., the school environment, colleagues, curricula used in the school, professional development agenda of the school or school district, etc.). What I heard also confirmed my experience teaching pre-service teachers: that changes inspired by the mathematics education reforms required teachers to learn and teach mathematics in new ways. This learning could come in many forms and venues. I was most interested in finding out from teachers themselves how they attempted to extend their mathematics education and how successful they thought they had been. I was very

interested in following up on student teachers I had supervised in the mid 1990s to learn of their experiences after they went out into their own classrooms. There were four areas I wanted to explore:

- How do the teachers describe the contexts in which they learned mathematics and learned to teach mathematics?
- How do they view themselves as learners and teachers of mathematics?
- What experiences, events, times, and opportunities do they identify as being especially significant in their development as learners and teachers of mathematics?
- How do they describe the ways those experiences affected their development as teachers of mathematics?

Though there has been extensive research during the 1980s and 1990s on the broad area of teachers' mathematics learning and teaching, there has been little analysis of teachers as mathematics learners from elementary school through their practice as professional teachers. In particular this has not been done with teachers using their own powers of self-reflection and self-reporting. Some research has been concerned with the kinds of knowledge specific to teaching and on deficits that teachers have in that knowledge (Ball & Wilson, 1990; Ma, 1999; Shulman,

1986). There has also been considerable interest in new courses that might better prepare prospective teachers for teaching mathematics for understanding and on specific professional development programs that might be effective in helping experienced teachers learn to teach mathematics for understanding (Ball, 1991; Ball & Lampert, 1998; Brown & Borko, 1992; Fennema, Carpenter, & Lamon, 1991; Sowder, Philipp, Armstrong, & Schappelle, 1998). Some studies have examined ways in which deficiencies in the mathematics content knowledge of teachers might act as an impediment to effective classroom teaching (Adams, 1998; Ball, 1991; Borko et al., 1992; Manouchehri, 1997; Simon, 1993). Other studies have followed teachers over time to see how teachers who have taken certain professional development courses have successfully changed the way they teach mathematics (Schifter, 1996a; Schifter, 1996b; Yaffee, 1996).

In the areas of teacher self-reporting, there are some narratives by and about teachers recounting their experiences as mathematics learners and teachers over the course of their own schooling and teaching (Schifter, 1996a; Schifter, 1996b; Schifter & Bastable, 1995). However, the existing narratives are written by or about teachers who participated in professional development

programs and workshops to learn more mathematics or to change the way they teach mathematics. The usefulness of these studies to the large number of teachers who have not been selected or who have not had the motivation to select themselves for these extensive in-service opportunities is limited. I feel more research is needed on the critical experiences underlying the mathematical development of these teachers.

This study seeks to add to our understanding of the circumstances that teachers find to be most important to their learning to teach mathematics. By not intentionally selecting for those teachers who have chosen serious supplementary mathematics training, this study will provide information about the other contextual issues that contribute to the development of mathematical content and pedagogical proficiency in the general teaching population.

Chapter II is a review of research on teachers learning to teach mathematics within the historical context of the mathematics education reform movement. Chapter III is a description of this study and the methods used in the research. Chapter IV is comprised of seven individual case narratives, and Chapter V is a cross-case analysis of the cases. Chapter VI is a presentation of some conclusions and recommendations that emerged from the findings.

Chapter II Literature Review

> It was generally assumed, and is still assumed by some today, that prospective elementary school teachers, and perhaps middle school teachers, learn all the mathematics they need to teach mathematics well during their own schooling. Recently this assumption has been seriously questioned. There is evidence of a vicious cycle in which too many prospective teachers enter college with insufficient understanding of school mathematics, have little college instruction focused on the mathematics they will teach, and then enter their classrooms inadequately prepared to teach mathematics to the following generations of students. (CBMS, 2001a, p.5)

The above quote is from *The Mathematical Education of Teachers* (MET), a report guided by the Conference Board of the Mathematical Sciences, and published cooperatively by the Mathematical Association of America and the American Mathematical Society. The document urges mathematicians and mathematics educators who teach future teachers to be more involved in the education of teachers and to rethink and reform the way mathematics is taught in college courses for prospective teachers. The document aims, also, at promoting greater collaborative efforts, on behalf of the education of teachers, among university mathematics departments, professional organizations, schools of education, school districts and state and national education departments (CBMS, 2001a).

The MET Report is one in a series of research studies and task force reports from the last two decades that have addressed questions and proposed solutions for improving the quality of mathematics education for K-16 students and the education of prospective K-12 teachers.

The current reforms in mathematics education emphasize the centrality of mathematical understanding. They espouse learning mathematics with understanding and teaching mathematics for understanding (Hiebert et al., 1997). The reform documents describe activities that contribute to and demonstrate mathematical understanding (Hiebert et al., 1997). For school age students, for example, understanding mathematics is described by the first four standards of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) which define mathematics as problem solving, effective communication, clear reasoning and making connections within areas of mathematics and between mathematics and other disciplines (NCTM, 1989).

If children are to learn mathematics by developing a conceptual understanding of the mathematical ideas they study, it follows that their teachers must have a deep conceptual understanding of the mathematics they teach. (Ball, 1991; CBMS, 2001b; Fennema & Franke, 1992; Grouws & Cebulla, 2000; Hiebert et al., 1997; Kilpatrick, Swafford,

& Findell, 2001; Ma, 1999; Shulman, 1986). Research about the mathematical knowledge of teachers, and the learning and teaching of mathematics for developing conceptual understanding, is the subject of this literature review.

I will first discuss selections from the relevant research on the kinds of knowledge teachers need for teaching in general and for teaching mathematics for conceptual understanding in particular. I will then discuss selections from the relevant research about the knowledge teachers have of mathematics and the knowledge for teaching mathematics as it develops during various points in their education and careers. These selections are key to my research topic and representative of the ideas in this field.

The Kinds of Knowledge Teachers Need for Teaching

In his 1985 Presidential address to the American Educational Research Association, Lee Shulman spoke about the kinds of knowledge that a teacher needs for teaching and how that knowledge differs from the kinds of knowledge needed by practitioners in a particular field. Shulman's speech grew out of research he and his colleagues had conducted to study how teachers construct knowledge in teaching. Their studies included observations and interviews of secondary teachers. Shulman had hypothesized

that "teachers draw from seven domains of knowledge - sets of cognitive schemata - as they plan and implement instruction: knowledge of subject matter, pedagogical content knowledge, knowledge of other content, knowledge of the curriculum, knowledge of learners, knowledge of educational aims, and general pedagogical knowledge" (Brown & Borko, 1992, p. 211).

Although they use different categories or names for types of knowledge, other researchers have also examined the complexity of teaching and the vast array of types of knowledge teachers need. *Developing Proficiency in Teaching Mathematics*, a chapter in *Adding it Up: Helping Children Learn Mathematics*, a review and synthesis of relevant research on the content and process of mathematics teaching and learning, presents three broad categories: knowledge of mathematics, knowledge of students, and knowledge of instructional practices (Kilpatrick et al., 2001).

The scope of this review is limited to the first category: knowledge of mathematics. While good teaching continually calls upon all three types of knowledge at once, the prospective teacher's field experience as a student teacher is primarily focused on developing the latter two types of knowledge: knowledge of students and knowledge of instructional practices. Indeed, the three

schools of education for which I have worked assume that students enter the student teaching semester with subject matter understanding in hand. However, in my experience, the reality is that many elementary student teachers struggle with the subject matter of the mathematics they are asked to teach. This experience led me to want to know more about subject matter knowledge and where and how people learn it.

In his presidential address, Shulman also focused mainly on subject matter knowledge and on what he coined as pedagogical content knowledge. He elaborated upon Joseph Schawab's distinctions between substantive and syntactic structures, substantive being the organization of facts and syntactic being "the set of ways in which truth or falsehood, validity or invalidity, are established" (Shulman, 1986, p. 9). Subject matter content knowledge "refers to the amount and organization of knowledge per se in the minds of the teachers. This knowledge goes beyond knowledge of the facts or concepts of a domain." It is an understanding of not just "how" something is done but "why" it works to do that (Shulman, 1986, p.9).

"How" one solves a mathematical problem and "why" one chooses a particular approach to a solution can be further

broken down into two types of knowledge: procedural and conceptual.

- Procedural understanding refers to mastery of computational skills and knowledge of procedures for identifying mathematical components, algorithms (rules and procedures for solving a computational problem), and definitions; it is knowing how to identify a problem and solve it correctly. For example, in division of fractions, following the rule of "invert and multiply" and carrying out that process would be an example of procedural knowledge (Eisenhart et al., 1993, p.9). In their review of research on teacher learning, Hiebert and Carpenter define procedural knowledge as a sequence of actions where there are minimal connections to create an internal representation of succeeding actions in the procedure (Hiebert & Carpenter, 1992).
- Conceptual understanding refers to knowledge of what is behind the procedure - refers to "the relationships and interconnections of ideas that explain and give meaning to mathematical procedures" (Hiebert & Carpenter, 1992). Referring back to the example of division of fractions, a person with conceptual knowledge would understand that dividing by a fraction asks either: 1) how many times one would find that fractional part in the original whole, or

2) how many whole units one would have if the original amount represented the fractional part by which one is dividing. A person with conceptual knowledge would also understand the nature of fractions, what it means to divide and what it means to divide fractions (Eisenhart et al., 1993). He/she would understand why the procedural rule "invert and multiply" yields a correct answer and why the answer makes sense. In contrast to procedural knowledge, conceptual knowledge is about relationships. It is knowledge that is "part of a network" (Hiebert & Carpenter, 1992, p.78).

Many studies of the 1980s and 1990s found that elementary and middle school teachers in the United States either lack procedural knowledge of the mathematics they teach or have a weak conceptual understanding of the procedures they use (Kilpatrick et al., 2001).

Pedagogical content knowledge, a term that Shulman coined, is defined as "subject matter knowledge for teaching … the ways of representing and formulating the subject that make it comprehensible to others - an understanding of what makes the learning of specific topics easy or difficult, of the conceptions and preconceptions that students of different ages and backgrounds bring with

them to the learning of those most frequently taught topics and lessons" (Shulman, 1986, p.9-10).

These differences between procedural knowledge and conceptual knowledge bring us back to the dilemma expressed in the opening quote to this chapter. A vicious cycle is perpetuated when teachers have only a procedural knowledge of mathematics and are therefore unable to teach their students in a manner that fosters conceptual understanding. It is important to reiterate that people who entered teacher education programs and began teaching in the 1990s were elementary and secondary students before the implementation of the reforms outlined in the 1989 *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). While there were many models of excellent mathematics instruction in the 1960s, 1970s and 1980s, most of this population of teachers studied mathematics in a much more procedural way than they are now asked to teach it. This phase difference between how teachers were taught and the proposed changes in the curriculum bedevils simple attempts at analysis.

Nevertheless, much of the research of the last two decades has made a case for intervention at various stages in teachers' education. Elementary teachers learn the mathematics they teach and how to teach it at several

points in time: their own elementary and secondary schooling, college, pre-service teacher education courses, student teaching, and while teaching. Keeping these stages in mind, from scores of articles I have read on this topic, I select the works of a few representative researchers to highlight issues of teacher knowledge and teacher learning during the different periods.

I have included the work of Deborah Ball because she was one of the earliest researchers to document deficits in teachers' subject matter content knowledge and pedagogical content knowledge of mathematics. Over the last 15 years, Ball has conducted numerous studies of many populations: elementary teachers, secondary teachers, pre-service teachers, experienced teachers, mathematics majors, and non-majors. Ball has been one of the strongest voices in the argument that subject matter should be "part of the equation" in teacher education.

I have included the comparative studies of Liping Ma and Stigler because they further document that teaching mathematics well requires more than procedural understanding.

I have included the work of Sharon Feiman-Nemser and colleagues of Hilda Borko and colleagues because they document the importance of subject matter content knowledge

in learning to teach and the problems student teachers face when they enter the classroom with insufficient subject matter knowledge. While mathematics methods classes might instill a desire to teach for understanding, student teachers have a difficult time implementing teaching for understanding without their own mathematical understanding.

I have included the review article by Putnam and Borko to give an overview of what an integrated approach to developing mathematics knowledge in context looks like. The work of Carpenter and his colleagues is an example of a professional development approach for which there is research evidence of success.

Teachers' Knowledge of Mathematics

Ball's documentation that teachers do not deeply understand the mathematics they teach began with her doctoral research using questions she designed and data she collected in 1986-1990 within a larger study conducted by the National Center for Research on Teacher Education at Michigan State University. *The Teacher Education and Learning to Teach Study* (TELT) was a multifaceted, multi-site, longitudinal study of both pre-service and in-service programs studying teachers who were learning to teach writing and mathematics (Education, 1988; NCRTL, 1991). The declared purpose of the study was to look "squarely at the

question of how formal teacher education bears on teacher learning" (NCRTL, 1991, p.1).

The TELT study examined eleven programs of both pre-service and in-service teacher education with regard to mathematics and writing. The purpose of the study was to look at teacher education programs, to describe the reasoning behind what teacher education programs did to help teachers learn to teach, and to describe the impact of the different approaches. Data were collected through questionnaires, interviews and observations. The study collected information about the knowledge with which teachers entered the programs (NCRTL, 1991).

One of Ball's contributions to the study was the design of a bank of mathematical questions that examined the teachers' procedural and conceptual understanding of the elementary mathematics topics taught in elementary school. Four of Ball's questions, later used in a comparative study by Liping Ma (Ma, 1999), probed teachers' conceptual understanding of subtraction with regrouping, multi-digit multiplication, division of a fraction by a fraction, and the relationship between perimeter and area. Ball analyzed responses, questionnaires, and interview data to look for indications of the teachers' ability to solve the problems she had posed, their ability to explain their

results and their ability to draw connections among mathematical concepts.

Some of Ball's data indicate that teachers sometimes lack even procedural understanding. In one study that included ten students preparing to teach elementary school and nine mathematics major and minors preparing to teach high school mathematics, Ball found that, when asked three questions about division (division by zero, division of fractions and division with algebra), not all of the prospective teachers could procedurally get the right answer and few could give mathematical explanations for what they did. The pre-service teachers also treated each division problem as a separate exercise without connecting the problems to one another. Ball found that the teachers had memorized how to do particular types of division problems, but that they did not have a conceptual understanding of the operation of division and they did not make connections between the various operations they performed on numbers (Ball, 1988).

Ball's analysis of other data reveals that even though many participants in her studies were able to procedurally solve a problem, their understanding of what they were actually doing in performing the procedure was quite fuzzy, even for very basic computational operations. For example,

in one study all of the participants could procedurally multiply the numbers and some of the participants used words like "add a zero" or "shift things over". Few participants, however, could articulate their actions beyond saying that the "zeros" were "place holders." They did not see the procedure was based on finding partial answers when multiplying. The participants did not have a firm grasp of place value and did not appreciate the importance of understanding place value as pre-requisite to understanding multi-digit multiplication (Ball, 1991).

Ball uses vignettes of teachers teaching in order to further illustrate the importance of subject matter understanding as necessary for teachers to teach for understanding. In her accounts of observations of three teachers teaching multi-digit multiplication in fourth grade, the teachers differed in their conceptual understanding of the traditional multiplication algorithm. Only one teacher, who had a deep conceptual understanding, was able to help students understand what the numbers represented. In the other two cases, the teachers merely told the students what to do procedurally and the students memorized a set of steps without any discussion of what the numbers they were manipulating represented (Ball, 1991).

Ball's research has done much to illustrate the complexities and demands of teaching mathematics for understanding and the limitations that teachers face when they do not have conceptual understanding of the mathematical ideas they are teaching.

Comparative Studies

Given the procedural nature of the teachers' understanding of mathematics in the United States, it should come as no surprise that comparison studies of mathematics instruction find that instruction in the United States is also procedural. Research studies that involve observations of classroom teaching show that throughout the 1960s, 1970s, 1980 and 1990s, there has been relatively little change in classroom practice and that teaching strategies in most mathematics classes can be described as rather traditional; that is, common classroom routines that include the teacher reviewing homework, a period of a teacher explanation of a new topic, a few problems worked out as illustrations, and time for students to work independently on the homework assignment (Hiebert, 1999).

In their analysis of video studies of United States mathematics classrooms for the Third International Mathematics and Science Study, Stigler and Hiebert found that most mathematics lessons revolved around procedures

and not concepts and that "96% of the time that students were doing seatwork they were practicing procedures they had been shown how to do" (Hiebert, 1999, p. 12). This picture of classroom instruction stands in contrast to mathematics classes in some of the other countries.

One aspect of classroom work these researchers examined related to whether teachers demonstrated how to solve a problem type before assigning problems for students to work on. There were big differences between the United States and other countries in the amount of demonstration used. Students had more opportunities to develop their own solution in the Japanese classrooms. Numerically, 63% of the Japanese lessons, 30% of the German lessons, and only 14% of the U.S. lessons included a level of work in which students created their own solution strategies (Stigler, Gallimore, & Hiebert, 2000).

Other comparative studies contrasted the conceptual understanding of United States teachers with that of teachers in other countries. Liping Ma asked four of Ball's questions to 72 Chinese teachers in 1998 and contrasted her results to Ball's 1987 results. Ma found that the Chinese teachers had conceptual understandings of elementary mathematics that were far more developed than that of the U.S. teachers (Ma, 1999). For example, in the division of

fractions problem, only nine U.S. teachers (43%) completed their computations and reached the correct answer (Ma, 1999, p.56). By contrast, none of the 72 Chinese teachers failed to solve the problem. Furthermore, while the U.S. teachers all solved the problem by using the common algorithm of "invert and multiply", the Chinese teachers proposed at least three other approaches (Ma, 1999, p. 61).

Ma outlined when and how the Chinese teachers develop profound understanding of fundamental mathematics. In China, teachers attend normal school to become teachers after the ninth grade. The Chinese teachers' understanding of mathematics was developed throughout their education. The Chinese teachers also reported that they studied teaching materials intensively. This includes studying the *Teaching and Learning Framework*, textbooks, and teachers' manuals. The teachers' manuals have much background information in them. The Chinese teachers also learn from colleagues and have a much more extensive period of mentoring and support during their first three to five years of teaching than is typical in the United States. The Chinese teachers also reported learning from their students and from doing mathematics (Ma, 1999, p 125-142).

In her conclusion, Ma reiterated that Chinese teachers learned subject matter knowledge in their schooling where

they attained mathematical competence, in their teacher preparation program where they connected this competence to a primary concern about teaching and learning school mathematics, and during their teaching careers where they develop a special teacher's knowledge which Ma calls "profound understanding of fundamental mathematics" (Ma, 1999, p. 145).

As Ma points out, data from the *Teacher Education and Learning to Teach Study* indicate that teacher education programs in the United States focus more on how to teach mathematics than on the mathematics content itself (Ma, 1999; NCRTL, 1991).

Mathematics Methods Courses in Pre-Service Teacher Education Programs

As an instructor of a course in teaching methods for teaching elementary mathematics, each semester I was faced with the dilemma of having students who did not understand the mathematics. It was a frustration both to me and to the prospective teachers that they had only a single course that was supposed to assure that they understood all of the mathematics topics they would teach, as well as the methods for teaching mathematics. Indeed, in a survey I did of fifteen graduate programs in elementary education in Massachusetts, not one program required students to take a

mathematics content course, and programs required only a single course in methods for teaching mathematics (unpublished data, 2000).

Feiman-Nemser and Remillard document the importance of content knowledge as necessary for teaching. They argue in *Perspectives on Learning to Teach* (Feiman-Nemser & Remillard, 1985) that insufficient attention has been given to content knowledge in teacher education programs:

> Traditionally not part of the teacher education curriculum, subject-matter knowledge is a central component of the content of learning to teach. Whatever else teachers need to know, they need to know their subjects. Current educational reforms have prompted renewed interest in teachers' subject-matter knowledge because they call for a kind of teaching that promotes powerful and flexible knowledge and understanding in students. (Feiman-Nemser & Remillard, 1995, p. 13)

Over the course of two decades of research on the experiences of pre-service and beginning teachers learning to teach, Feiman-Nemser and colleagues repeatedly document that poor subject matter preparation is not remedied in pre-service course work. In *Knowledge Use in Learning to Teach,* a research project coordinated by Feiman-Nemser, researchers studied the transition from coursework to student teaching by following prospective teachers in two different programs through a year of coursework and through their student teaching semester. In two case studies of two teachers each, one from the first year (Feiman-Nemser &

Buchmann, 1985), and the other during their student teaching experience (Feiman-Nemser & Buchmann, 1986), Feiman-Nemser and Buchmann find that student teachers require more explicit instruction, help and guidance from their supervisors and cooperating teachers. These are needed for them to develop new ways of thinking about how to present subject matter to students. In both case studies there is an example of learning to teach mathematics that highlights this point.

Their most vivid portrayal of the need for guidance comes in the actual narratives. For example, one narrative about "Janice" chronicles her thoughts about teaching math which derived in part from her experience in tutoring her brother in sixth grade mathematics. This experience led Janice to realize "'you have to really know math in order to be able to teach math'" (Feiman-Nemser & Buchmann, 1985, p. 10). Yet, Janice said that she did not learn math in her math methods class, but learned "'different ways of teaching math to different age groups'" (Feiman-Nemser & Buchmann, 1985, p. 10). Janice plans to rely on her own experiences learning mathematics. She also expresses frustration that she will have to rely on using textbooks to teach mathematics when a reliance on textbooks runs counter to the philosophy of education she has developed in

her teacher education program. The authors point out the missed opportunity of the teacher education program in helping Janice learn new ways of teaching mathematics (Feiman-Nemser & Buchmann, 1985, p.11).

In another case, "Molly", one of the student teachers in the case study, gives an example of the rules she told her students for playing a game to develop concepts of place-value in a mathematics lesson. The authors point out that Molly does not "make a distinction between those [rules] bearing on behavior and those bearing on mathematics" (Feiman-Nemser & Buchmann, 1986, p. 29), giving equal weight to telling students to turn ten chips in for one chip and where to roll dice (Feiman-Nemser & Buchmann, 1986, p. 29). This again points to a potential weakness in deep understanding of content knowledge. Molly is reacting as if she were following a recipe for teaching (she may in fact have been following a scripted lesson plan) instead of deriving her teaching from her own understanding. These examples raise the question of where different teachers are to develop their content proficiency and how they will be helped to avoid the pitfalls that Janice and Molly portray.

Borko and her co-researchers of the *Learning to Teach* Study, in a large longitudinal study on the complexities of

learning to teach, also identify the issue of what constitutes weak subject knowledge and how such knowledge impacts the work of the student teacher. In one study, Borko et al. demonstrate how weak subject knowledge impedes a student teacher's ability to translate what he/she learned in a mathematics methods course into effective teaching. In studies of pre-service teachers, Borko has written about the difficulties of student teachers having to navigate mixed roles (student and teacher), which contain mixed emphases: theoretical in their methods classes versus practical in their student teaching experience. Teachers also have had exposure to different methods of teaching, which include the way they were taught as K-12 students, the way they were taught at the university, and the way they see their co-operating teachers teach.

One of Borko's articles on research data gathered in the Learning to Teach Project, *Learning to Teach Hard Mathematics: Do Novice Teachers and Their Instructors Give up too Easily?* (Borko et al., 1992), is particularly relevant. The research highlighted the differences between subject matter content knowledge and pedagogical content knowledge, and the differences between the kinds of understandings needed to do mathematics and to teach

mathematics. The article centers on an analysis of classroom observations of a sixth grade student teacher, Ms. Daniels, who is in her final semester of four student teaching placements. Ms. Daniels was a math concentrator in an undergraduate teacher education program. Because she had taken two years of upper-level mathematics courses in college, Ms. Daniels was allowed to test out of the content course for elementary teachers. She did, however, take a course in methods for teaching mathematics prior to student teaching. Despite her background, the researchers found that Ms. Daniels entered the "student teaching year with only a rote understanding of division of fractions and no knowledge of representations that might enable her to teach the topic except by demonstration of the algorithm" (Borko et al., 1992, p. 207). Ms. Daniels believed that students' mathematics instruction should center on real life problems. Yet, despite having studied division of fractions in the mathematics methods course, Ms. Daniels was unable to answer a student's conceptual question as to why the rule "invert and multiply" worked in dividing one fraction by another, and Ms. Daniels inaccurately constructed a "real life" example to illustrate division of fractions (Borko et al., 1992, p. 207).

The authors of the study point out that the courses Ms. Daniels took did not prepare her to teach fractions. The university courses stressed rote learning. They assumed a procedural proficiency with fractions but not the nature or properties of fractions. Based on their analysis, the authors propose that universities should provide better opportunities for students to strengthen both their subject matter knowledge and pedagogical content knowledge of the mathematics they will be teaching (Borko et al., 1992).

Furthering Mathematics Knowledge for In-service Teachers

What about the teachers who are already teaching? One should ask whether they have an opportunity to go back and learn mathematics in a new way in order to bring a greater subject matter understanding into their classroom.

Recent learning theories have stressed the importance of the context and situation in which people learn. In a recent article, *What do New Views of Knowledge and Thinking Have to Say about Research on Teacher Learning?* (Putnam & Borko, 2000), Putnam and Borko review current research about teacher learning which has come to be known as the "situative perspective" of knowledge, thinking and learning. This research investigated the idea that knowledge, thinking, and learning are situated in one's

experience. Current researchers such as Brown et al., Green, Lave and Wenger, "posit that the physical and social contexts in which an activity takes place are an integral part of the activity, and that the activity is an integral part of the learning that takes place within it" (Putnam & Borko, 2000, p 4).

Looking at the environments of learning and teaching as situated has implications for teacher pre-service and in-service education. Putnam and Borko consider three issues: where to situate teachers' learning experiences; the nature of discourse in communities for teaching and teacher learning; and the importance of tools in teachers' work. Pre-service and in-service teachers often say that the best kind of learning is the learning they do within their own classrooms. However, there is an argument that if the goal is to get teachers to see mathematics in new ways, it is best for them to have experiences with mathematics in settings outside their classrooms. Teachers can gain a fresh perspective on their teaching by describing their classroom experiences and discussing specific occurrences with other teachers (Putnam & Borko, 2000).

Putnam and Borko describe various designs and learning environments for professional development programs. They

conclude that the appropriate format and content depend on the goals for the program. For example, when the goal is to enact specific instructional practices, experiences such as coaching or modeling situated in teachers' own classrooms may be a good approach. A few programs, for example SummerMath for Teachers and Cognitively Guided Instruction, have effectively combined the use of summer institutes with in-class professional development during the following school year to help teachers integrate their own learning into their classroom practice (Putnam & Borko, 2000, p 7). Summer institutes and courses, as well as institutes held during the school year, employ a variety of means by which teachers can examine their own understanding of mathematics and their knowledge of student learning. Some of these are: doing mathematical activities themselves; examining student work; using cases (written, video, and hypermedia) in which they can share common experiences of what happens in a classroom.

An increasingly prominent realization is that teacher learning is a lifelong process and that professional development programs are best when they engage teachers in their classrooms (Kilpatrick et al., 2001). *Developing Proficiency in Teaching Mathematics* gives four examples of integrative professional development programs that focus

respectively on mathematics, student thinking, case study and lesson study (Kilpatrick et al., 2001).

An example of a successful professional development program is Cognitive Guided Instruction (CGI). CGI not only situates learning in the classroom but also is a good example of teacher learning that fosters teachers' building of new constructs from what they already know. CGI was developed when researchers at the Wisconsin Center for Educational Research first conducted studies on how children learn mathematics. They found that young children came to school with a natural curiosity and an informal and intuitive sense, which enabled them to solve math problems at a level far beyond that of which teachers thought the students capable. The student solution methods use concrete materials to model the situation presented in the problem (Carpenter, Fennema, Franke, Levi, & Empson, 1999). The researchers became interested in what knowledge teachers brought to mathematical instruction and whether teachers' understandings of their students' mathematical knowledge affected their teaching. In a series of studies, the researchers found that "learning to understand the development of children's mathematical thinking could lead to fundamental changes in teachers' beliefs and practices" (Carpenter et al., 1999, p. 105).

In the first study, the group measured forty first grade teachers' knowledge of mathematical understanding of different addition and subtraction problem types, strategies children use in solving such problems, and problem solving abilities of individual students in their classrooms (Peterson, Fennema, & Carpenter, 1991, p.51-56). They found teachers had intuitive knowledge of children's mathematical thinking but were uncertain about problem solving strategies children use and the relative difficulty of certain problem types (Peterson et al., 1991, p.54-56). In a follow-up study, they created a four-week summer workshop (CGI), which they offered to twenty teachers randomly selected from the forty first-grade teachers (Peterson et al., 1991, p.64). The workshop met five hours a day, four days a week for four weeks. The CGI workshop participants viewed and discussed videotapes of children's solution strategies and read and discussed papers on how children solve problems. Teachers in the workshop worked in groups to plan lessons for students. Some of the workshop time was unstructured to give teachers time for reading, viewing additional videotapes, talking with one another, examining textbooks and other teaching materials or planning for the upcoming year. While there were no formal written assignments, it was suggested that teachers plan a

unit and make an instructional plan for the year. The teachers were not told specific teaching strategies to use (Peterson et al., 1991, p.64 - 68).

In the following year, researchers examined the teachers' classrooms and found that the teachers in the CGI workshop posed problems to students more often, listened more carefully to students, encouraged multiple solutions to problems, engaged the students in active group participation and conducted more discussions on problem solving than did teachers who had not taken the CGI workshop (Peterson et al., 1991, p.68). At the end of the year, a number of instruments used initially to measure attitudes toward and knowledge of mathematics teaching were re-administered to the forty teachers in the original study. The CGI teachers were more knowledgeable of the problem solving strategies their individual students used, and were more accurate in their assessments of individual student knowledge, than were teachers who had not taken the workshop (Peterson et al., 1991, p.67-68). Overall, the results of the series of studies were that increasing teachers' knowledge of children's mathematical thinking has a positive affect on teachers' instructional decisions and student learning (Carpenter et al., 1999).

Conclusion

The mission of the mathematics education community of the 1980s and 1990s was to make major changes in the ways mathematics is taught to children. This was to be done in line with new findings of NCTM and others about students' understanding of mathematics. In addition, the mission was to prepare teachers to teach mathematics in a way often different from how they were themselves taught mathematics. For school children, mathematics education moved away from rote procedural learning to education that fostered conceptual understanding and an appreciation that mathematics consists of problem solving, reasoning, communication and connections between areas of mathematics and between mathematics and other disciplines. Teaching mathematics in this manner requires a different kind of knowledge than is required if teachers merely show students how to apply manipulations and operations which the students memorize. Research of the late 1980s and early 1990s shows that many elementary and middle school teachers do not have a conceptual understanding of the mathematics they teach and sometimes lack even a procedural understanding. This lack of conceptual understanding stands as an impediment to teaching for understanding. At every juncture, elementary school through teacher

professional development programs, there is an opportunity to intervene and help teachers develop their own mathematical understanding.

Research in this whole area is complicated by various factors, chief among them being the diversity of educational, training and teaching experiences. As mathematics curricula have moved in the direction of problem solving and teaching for understanding, teachers have only slowly changed their aggregate experience in these areas. True control groups are hard to define and might not be valuable in understanding this issue. Nevertheless, the studies all point to deficiencies and opportunities at every level. The metaphor of mathematical content knowledge as a vortex that can either spiral up or down, influenced incrementally by the teachers' own experiences in elementary school, high school, college, pre-service, student teaching, in-service, and classroom teaching is an apt one.

With all this in mind, I embarked on this study to discover what seven teachers found to be significant to their development as mathematics teachers. These seven teachers were representative of the population of teachers who were educated in mathematics procedurally and who were expected to teach mathematics for conceptual understanding.

I embarked on the study to see whether the teachers' experiences matched the findings of other researchers.

Chapter III Methods

Overview and Rationale for Using Qualitative Inquiry

The purpose of this study is to reflect the views of seven elementary teachers entering the profession in the mid 1990s, as they relate the significant influences in their learning mathematics and learning to teach mathematics. A qualitative method of inquiry is appropriate for eliciting teachers' own documentary histories. The goals of this study are aptly suited to qualitative inquiry as described by Patton in *Qualitative Research* (Patton, 1980).

- This is a naturalistic inquiry studying a real-world situation. I aim not to control the situation but to uncover the teachers' descriptions of the phenomena of their learning to teach mathematics.
- The categories of what experiences are significant to teachers emerge from the details and specifics of what the teachers say.
- The data are in the form of "detailed, thick descriptions" reported in the teachers' own voices and the narratives use direct quotations to "capture their personal perspectives of experiences" (Patton, 1980).
- As a researcher, my role is what Ely et al. describe as the "privileged observer": "someone who is known

and trusted and given easy access to information about the context" (Ely, Anzul, Friedman, Garner, & Steinmetz, 1991).

- The questions in this study are open-ended and genuine, without a preconceived theory. Rather, an area of inquiry, a set of questions, is explored.
- The choices that teachers make about which experiences to relate are informative in themselves and reveal which experiences are most important to them.

In this study I report what teachers say about experiences at various times in their lives, during their own schooling, teacher preparation and while teaching. Case narratives are an appropriate format for analysis of the teachers' stories. Themes from the individual narratives of the seven participants form the basis for a cross-case analysis considering all seven participants together.

Process: Getting Ready for the Study

Participants were recruited for this study from the pool of teachers I had supervised when they were student teachers. These teachers had entered elementary school teaching in the mid 1990s amid major changes in elementary school mathematics education. I chose teachers who had been my former students because I had already established a

relationship with them, which was built on trust; yet, I no longer had any formal or supervisory role in the school districts in which they worked. I believed this combination of trust on the one hand, and lack of current authority on the other hand would allow the teachers to talk openly with me. Furthermore, I had been an integral part of their education at one phase of their development, student teaching. I was interested in their perceptions of their experiences during this teacher preparation phase of their development as well as their perceptions of their own schooling and their teaching experiences.

The participants had done their student teaching at an elementary school, which I will call the Fifth Street School, which served as a professional development site for placing some of the teacher education students at General University (also a pseudonym). I was the university supervisor for most of the students, including the participants, at this particular elementary school site for a five-year period. I was very familiar with the context of their student teaching experience with regards to the culture and philosophy of the teacher education program they attended and the school site at which they worked. I was interested in the circumstances in which teachers learn and teach, and I anticipated that those circumstances would

have an important bearing on their mathematics education as teachers.

The characteristics of the school site and university program and the structure of their relationship with one another are discussed further below.

Recruiting Participants

Of the forty-seven students with whom I had worked in this program, I was able to locate current information for thirty-two individuals. I spoke with thirty of them. I asked them whether they were currently teaching mathematics as part of their day and whether they might consider participating in a study I was planning to do on elementary teachers learning to teach mathematics. I explained that I was seeking participants for a study that would include my observing them teach mathematics and my interviewing them about their learning mathematics and learning to teach mathematics. From this group of thirty, seven teachers volunteered, obtained permission from their principals, and scheduled a classroom observation in spring 2001.

Participants' Background

Of the seven teachers in the study, there is one man and six women, ranging in age from early thirties to early forties. At the time of the study, the teachers were teaching at the following grade levels: one at

kindergarten, two at second grade, two at third grade, one at fifth grade, and one at seventh grade mathematics. Five teachers worked in public schools and two in private schools. Of the five public school teachers, two taught in a big city and three in suburban schools. (See Summary of Teaching Experiences in Appendix).

The participants in the study represent many characteristics of the population of prospective teachers who had come through conventional teaching programs. All had been elementary and secondary students before the publication and implementation of the 1989 NCTM Standards. They had earned bachelor degrees at a variety of institutions and had graduated college between the years 1989 and 1993. Two participants had started college directly out of high school, but did not graduate directly, and took a final course or two in 1990s because they wanted to go to graduate school in a teacher education program.

All participants had taken at least one semester of college level mathematics (or, in one case, one semester of computer science) thereby satisfying the mathematics requirement for an undergraduate degree at their institutions as well as fulfilling the prerequisite requirement in mathematics for entry into the graduate teacher education program. None of the participants were

required to take additional mathematics courses while in the graduate education program and none chose to do so. The participants did, however, take a required one-semester course in elementary mathematics teaching taught in the school of education. The curriculum of this mathematics education course was influenced by the publication of the NCTM *Curriculum and Evaluation Standards* and also included study of the Massachusetts mathematics frameworks.

The participants began the two-year combined teacher-certification and masters degree program between the years 1993 and 1997 and graduated between the years 1996 and 1999. They did their student teaching at Fifth Street School for a fifteen-week single semester during the years 1996 through 1999. Student teaching was accompanied by a two-hour-long weekly student teaching seminar held at the elementary school site. As the university supervisor and seminar instructor working in the same school each semester, I was also very familiar with the cooperating practitioners, the classroom teachers in whose classrooms they did their student teaching. As supervisor, I observed each student teacher teaching at least six times during the semester, including at least one mathematics lesson. Each observation was followed by a 45 minute to one-hour post-observation conference with the student teacher and

sometimes with the cooperating practitioner. I kept notes on all observations and meetings. I also had at least two other meetings with the student teachers and their cooperating practitioners at the school.

The prevailing philosophy of mathematics instruction in both the teacher education program's course for teaching mathematics and the classrooms at the student teaching site were consistent with that of the NCTM Standards. They de-emphasized memorization of procedures and algorithms and emphasized the development of reasoning, problem solving, communication, and forming connections within strands of mathematics and between mathematics and other subjects.

Teacher Education Program

General University, an institution in a large metropolitan area, offers teacher certification programs to both graduate and undergraduate students. Teacher education students at General University did their student teaching for the full semester, the final semester of their program, after having completed all or most of their other coursework. During the semester prior to student teaching, prospective teachers documented completion of their pre-requisites and met with the placement officer for a student teaching assignment. The placement procedure at General University often included student teachers' visiting

potential school sites, which was the case at Fifth Street School. During the student teaching semester, the student teacher worked in the classroom of a teacher called the cooperating teacher and was visited at least six times by a university supervisor. (Although the state required only a six-week student teaching assignment for provisional certification, General University's program specified a fourteen-week student teaching assignment.) During the student teaching semester, students took an accompanying seminar course that was taught by the supervisor.

Student Teaching School Site

Fifth Street School is a public school with an excellent reputation. The school has about 350 students distributed among grades K-8 school. The population of students and staff at the school is racially, ethnically, economically, linguistically, and culturally diverse. The teaching staff was an experienced staff with most of the teachers having entered the profession in the late 1960s or early 1970s. The philosophy of education centered on respect for the students, building a community, student-centered activities, inquiry methods, group-work with individual accountability, and use of teacher prepared curricula. There was a low turnover rate of staff: there were only two teacher changes in the twenty classrooms

during my five years at the school. The principal and many staff members at the school had been together for fifteen or twenty years. The school operated with a spirit of teamwork and cooperation among the staff with active participation by parents and community volunteers.

The mathematics curricula used in the classrooms were either teacher-created or used a standards-based curriculum that "emphasizes reading, problem-solving, everyday applications, and the use of calculators, computers, and other technologies" (Everyday Mathematics for grades 1-5 and Connected Mathematics in grades 6-8).

There were some special features about the relationship between Fifth Street School and General University. One of the elements of reform in teacher education the late 1980s and the 1990s promoted cooperation among schools and teacher education programs in the education of prospective teachers. Formal arrangements called Professional Development Schools fostered varying levels of involvement between the two institutions in training teachers. Fifth Street School and General University had such a professional development relationship.

Each month, a team of six teachers at the school, a faculty member at the University, and a supervisor of

student teachers at the University met to discuss the needs of prospective teachers and the ways to improve the education of student teachers both at Fifth Street School at General University. Out of this dialogue, emerged the following defining features of this particular professional development relationship: 1) a cluster of five to eight student teachers were placed at the school site each semester, placements were made by matching student teachers and cooperating practitioners based on input from both of them after potential student teachers observed and talked with the teachers 2) the weekly student teaching seminar course was given at the school site instead of the university, 3) teacher input was included in designing the curricula of the student teaching seminar course and the university course in mathematics teaching, and 4) the six teachers co-taught the student teaching seminar with the university supervisor (the six teachers each taught one session of the redesigned course).

Data Collection

Three methods of data collection were employed in this study: interviews, observations and written reflections of the teachers.

Interviews

The primary source of information on the teachers' perceptions of their learning mathematics and learning to teach mathematics came from individual interviews conducted in the summer of 2001. Each interview lasted from seventy-five minutes to two hours and probed the participant's experiences and feelings as a student of mathematics during the years of elementary school, secondary school, college, post-graduate teacher preparation program, and the pre-service student teaching semester, as well as during their in-service teaching and professional development experiences. Each interview was audio-taped and transcribed. My familiarity with the various cooperating teachers and with the mathematics programs used in the various classrooms in which the student teacher worked facilitated the communication of information during the interview and gave me a context for listening to what the participants had to say about their student teaching experience. The observations I made of their teaching also gave me a context for understanding what they said at the interviews.

To confirm the face validity of the data, I conducted a telephone interview with each participant six months after the first interview. With participants' permission,

I typed their responses as they gave them. I found that each participant mentioned the same significant experiences as in the original interviews and frequently used the same phrases to describe their experiences. At the second interview, I also asked participants about their current situation teaching mathematics. Two participants introduced information about an experience that had happened during the six months period that they found to be significant to their development as a teacher of mathematics.

Observations

I did the initial observation of a mathematics lesson as one of the six lessons I observed during each participant's student teaching semester. The notes of these observations included records of the participants in their student teaching classes, during post-observation conferences with them, and during discussions with the participant and the then cooperating teacher.

A second set of teacher observations was conducted in the spring semester in May or early June at the end of the school year, 2001. By my visiting late in the school year, I had allowed time for teachers to have worked out their classroom relationships with the students for a long period of time (semester or year). Also by this time of the year,

teachers were very familiar with the curricula they were using and had well established routines for classroom instruction. Thus, by choosing the late spring to make the observations I could focus attention on the teachers' teaching of mathematics and not on general teaching skills.

The observations provided a context in which the teachers could talk with me about their development as learners and teachers of mathematics. My conducting observations at two points in time gave me a perspective with which to understand the teachers' descriptions of their development over time. The observations also allowed me to note areas of strength and weakness in the teachers' subject matter knowledge and gave meaning to what the teachers had to say about their subject matter knowledge and experiences in studying different mathematical concepts. The observations made the interviews more vivid. Similarly, what I learned from the interviews allowed me to re-examine my perceptions of the observations.

Data Analysis

This study is a narrative case study of each of the seven participants followed by a cross-case analysis of themes that emerged from examining all seven cases together. The case narrative of each participant was based on all of the data I collected regarding that participant.

I analyzed the data by listening to each audio-taped interview several times while reading the transcripts and taking notes. Referring to the notes from the interviews and observations, I crafted a narrative for each teacher, which recorded his or her account of what was significant in his or her learning mathematics and how to teach mathematics.

The participants differ in their past experiences and relationships to mathematics learning and in the experiences they have had as teachers of mathematics. Yet it is important to see what more general lessons we can learn from their individual stories. So, after writing the seven case studies, I made a cross-participant analysis to discover what themes recur and what factors the participants report as having been influential in their development of teachers of mathematics.

Chapter IV Narratives

Each of the seven narratives in this chapter is presented in roughly chronological in order to give the sequence of significant experiences and to show how they build upon one another. The names of all people and all institutions are pseudonyms.

Beth

> And I think the thing that opened my eyes [about teaching math] was actually in the science class Don Smith teaches, and he's great. He was awesome. That was my first semester… and [I] was talking to somebody, looking at the education books [who said] "Oh, you've got to take a class with Don Smith." So I did. [During one particular class - which I will describe…] I remember thinking, "Wow, if this is how you can teach math, then maybe it wouldn't be so bad."

Beth, a woman in her mid-thirties, presently teaches mathematics in the seventh grade in a large suburban middle school. After graduating college in 1989, with a major in mathematics and a minor in studio art, she worked for non-profit organizations for seven years before entering the elementary teaching program in 1996 with plans to teach in grades five or six. Central to her descriptions of the contexts in which she learned mathematics, and learned to teach mathematics, are her comments about her teachers. Beth identified three people who were especially significant in her development as a mathematics teacher: Dr. Jones, her mathematics professor and undergraduate

advisor; Dr. Smith, the science education professor in her teacher education program; and Ms. Fox, her cooperating teacher during her student teaching experience.

As Beth relates, the styles of these three teachers contrast greatly with the styles of her teachers during much of her other schooling. These three teachers share these common characteristics: 1) each values developing student thinking in their teaching; 2) each employs teaching strategies that are active, engaging and visual; 3) each promotes student independence; and 4) each asks questions and develops exams and activities that require students to extend their knowledge, not merely repeat back what the teacher did.

Beth's early experiences help provide a context in which to understand what about these teachers appealed to her. She grew up in the 1970s and 1980s in a mid-sized working class city that she describes as a "rough town". She attended "very traditional" public schools in which she was always in the "highest track." The only math activity she mentions from elementary school was the use of a boxed set of cardboard numbers and operation symbols with which to make equations.

Beth mentions no elementary or secondary teacher as being inspiring and gives little more than a list of

subjects when talking about her K-12 experience. Starting in the eighth grade, she took algebra, geometry, algebra II, trigonometry and calculus. She says she "loved algebra" and geometry is her "favorite subject." She describes herself as a "very visual person" who "naturally sees pictures," and says she also has a passion for doing visual art – especially sculpture. She reports that she particularly enjoyed Janet Jones's abstract algebra course in college because Dr. Jones used visual presentations.

She recalls her last two years of high school as ones in which she did minimal work in her math classes. She says she was "offended" at getting an A in trigonometry when she "didn't deserve it". She totally "slacked-off" in calculus and actually failed the course. Beth's uninspiring high school math courses were counterbalanced by a couple of positive mathematics experiences outside of class. One experience was studying math in a high school program at a local elite college. There, on Saturdays, she studied three-dimensional polyhedrons in a "hands on", "really fun course", taught by an undergraduate student at the college. Through the same program she got her first experience with teaching when, as high school seniors, she and a friend co-taught a course in algebra to middle school students.

The second positive experience outside of class was that Beth and another student tied for the highest score on a mathematics competition exam in their high school. Beth recalls the exam as a test in problem solving ability. She says, "They gave you information and you had to solve problems, which I'm much better at." Her test score boosted her confidence.

After graduating high school in 1985, Beth immediately went to a small, elite, private women's college where she took calculus her freshman year. She says, "I decided I was going to take calculus and pass it if it killed me." She says she worked hard, enjoyed the course, and was pleased that she got a B in this "hard course." She kept taking mathematics courses but says, "[I] didn't consider myself a math person, even though I majored in it." She says she became a mathematics major "by default," primarily because she did not want to major in a subject requiring much writing. She explains that she entered college feeling she was less well prepared than were her classmates, who had mostly graduated from private schools or public schools in wealthy suburbs. She adds, "I was very shy and writing was very stressful because I felt like I was being judged. With math, I could just sort of do the math, it's not very personal".

Beth also says that she really liked going to a women's college and notes that her college graduates a disproportionately high number of math and science majors. She took most of her math classes with the same students, the other women majoring in math, and she describes the classes as non-competitive, supportive and conducive to good discussions.

From the more than twelve mathematics courses Beth took in college, she singles out Dr. Jones as being an inspirational teacher and influential person in both her learning math and in her learning to teach math. Beth describes Jones as a "nurturing" teacher who taught the course in a way that allowed Beth to "visualize" the subject matter. Beth says that Professor Jones's assignments and tests required students to think:

> I just worked so hard in her class. I think her exams were more an extension of what we learned rather than a test of what we learned. We had to – like I remember she gave us – a lot of the exams were take-home exams. So the problems would be harder or different. So we'd really have to think about it rather than just solve them.

Beth mentions feeling excited when she was able to solve these challenging problems. These activities reinforced Beth's view of herself as a mathematics "problem solver" and not just a computational machine. She says now that she is a teacher she "analyze[s] things from a

teacher's perspective." She names Janet Jones as her model of a teacher "who teaches math in the manner [I] want to teach."

Beth also reports being excited by the things she learned in Professor Smith's class in graduate school. Once in a teacher education program, Beth said she focused on observing Smith's teaching style as well as the subject matter of the course. She says:

> Once I got to the Teacher Education, I really started observing the teachers teaching, and I started observing the way I learned. And I think in a way the process of doing that was more valuable than what I was learning, though what I was learning was important, too.

In a paper she wrote while in the teacher education program, Beth described Smith's class in the following way:

> One common theme of our class is that students should be given freedom to experiment on their own. Usually Don gives an introduction to the experiment, lets people get their materials and then get to work. He is available for questions but stays out of the way until the discussion at the end (unpublished paper, October, 1996).

Beth says that she first got to put into practice the teaching strategies she wanted to use when she did her student teaching. She did her student teaching with two veteran teachers, Susan and Mark, who shared a sixth grade classroom, (each teaching half the week); Beth says she appreciated getting to observe two different teachers. Beth describes two different types of math lessons in this

class: in one, the students were given open-ended word problems, which they worked on individually or in small groups, coming together for discussions. In the other, lessons were part of multi-lesson units in which there was usually a brief introduction, an exploratory activity period (often using manipulative materials) in which students usually worked in groups, and a wrap-up discussion period when students explained their solutions to one another. Beth appreciates the emphasis on helping students recognize that there are multiple ways of finding a given solution and that some problems have more than one solution. She contrasts this with her own schooling and says, "The way I learned math was that there was one answer."

After the first couple of weeks of student teaching, Beth taught her own math group of eleven students. I observed her teach a lesson in which the students used one-inch cube blocks to build rectangular prisms. For a given number of cubes, students constructed as many prisms as they could, recording the volume and surface area of each. During the wrap-up, students discussed the relationship of surface area to volume.

Beth's positive experiences studying and teaching mathematics in her pre-service program led her to consider

teaching middle school math when she graduated. Since graduation, she has had two jobs teaching seventh grade mathematics in middle school. The first position was in a school district in what Beth describes as multi-ethnic, multi-racial, predominantly poor and working-class community. Beth says the principal "was great". He tolerated no racism and set the tone for a harmonious atmosphere in the school. The teaching staff was comprised of mostly older teachers who were happy to welcome Beth as a new teacher in their school. While there was no official mentoring program, Beth says she felt "taken care of" and that one teacher in particular "took [her] under her wing". The principal, a former math teacher, was supportive of Beth's trying new curricula. After Beth researched curricula and chose to use *Connected Mathematics Project*, an inquiry-based middle-school mathematics curriculum comprised of several units a term, the principal supported her decision and ordered 150 copies of three units to be used in the seventh grade the following year. The district for which Beth worked offered professional development workshops on Saturday mornings on a variety of topics. Beth attended the only workshop specific to mathematics, a two-day workshop on strategies for teaching specific

mathematical topics. Beth says that in her teaching she uses some of the ideas from the workshop.

Beth relates many features she liked about the first school at which she taught and the working environment of the school. She says that she liked the diversity of the school population and the atmosphere of respect for differences that the principal promoted. Beth says that she identifies with the teachers, most of whom had grown up as good students in the public schools in the city in which they were now teaching, a working class community similar to the one in which she had grown up. She describes an "intellectual environment" among the staff and says she appreciated the commitment of the teachers to the students, the school and the community that arose, perhaps "because the teachers had to really work hard no matter what."

Beth says she found the school environment conducive to learning for both her students and herself. She says the environment and the supportive principal and staff made her happy to work at the school despite a two-hour daily commute. With the enthusiastic backing of the principal, Beth had planned to return to the school the following year. However, when a bureaucratic snafu at the district level made it uncertain as to whether she would have a position at that school, she looked for another job. By

the time her former district had offered her a job on the last day of the school year, Beth had already accepted her current position. She now teaches seventh grade at the only middle school in a middle-class, moderately sized city which is less racially and ethnically diverse than was her first school.

Beth contrasts the new school environment to the old. She describes the new district as a "very traditional school system" in which many of the students and teachers are not interested in trying new approaches to learning mathematics. Her first year at the school was the last year of an outgoing principal's tenure. He had imposed a house system at the school over the objections of the staff. Beth says she thinks that many of the teachers, having grown up locally and gone to a nearby college in this homogeneous community have a more "narrow" perspective on life and teaching than her former colleagues.

Beth has about 30 students in her classes and has two regular classes, one honors class and one below average class. Beth says that she "does not agree with having levels in the seventh grade."

Last year the school very slowly introduced the *Connected Mathematics Project* curriculum by purchasing one class set of thirty books for each of three different units

(a total of ninety books) at each grade level. Teachers were told that they could use as many of the investigations from as many of the three books as they chose to use along with the standard textbook series. As there are eight units in the *Connected Mathematics* series written for the seventh grade and there are over 250 seventh grade students at the school, the purchase of ninety books meant that the new curriculum could be used only as an add-on and not as a way to overhaul the mathematics curriculum in the school. Also, the school provided no professional development or support to accompany the new curricula, and decisions on which units to purchase were left to the staff at each grade level, without any cross-grade coordination. For example, there was no effort to order the unit on area and perimeter in the sixth grade and the unit on volume and surface area in the seventh grade. In fact, Beth reports that there is very little time or opportunity to meet with teachers across grade levels. Beth says group meetings of math faculty with the district-wide math coordinator are too short, too infrequent, and poorly timed to be conducive to interaction among participants (fifteen minutes bimonthly before school). Beth says she does talk informally with the other two seventh-grade teachers but

that she is pretty much on her own as to what she does in her classroom.

Beth believes that she is still learning and developing as a teacher. She says that she sometimes finds her "classes tend to be kind of traditional, more traditional than I would expect that they would be." She uses her experiences and student feedback to help her make changes in her teaching in subsequent years. At the end of last year, Beth surveyed her students to ask their opinions about the two curricula. They had used *Connected Mathematics* for one five-week unit and the standard text for the rest of the school year. Beth states that the students split down the middle in their selections of the curriculum they preferred:

> And I think the students who like to think about things did enjoy the *Connected Math*. But the students who were just - had been good students, they were good at repeating, you know, throwing back what I taught them, they liked the textbook better because that's the way the textbook works.

As to her own preference, Beth mentions both Jones and Fox and says she prefers an approach that helps children learn to think. Although the school has now added another *Connected Mathematics* unit, which all seventh grade teachers will use, Beth finds it less than ideal to teach a hybrid program and to use a text she doesn't like. Beth says she also finds it difficult to teach this new

curriculum in her current setting for several reasons: 1) forty-five minute periods are too short for the activities; 2) classes of 30 students are too large for her to circulate among all of the students as much as she would like; and 3) some students are resistant. On this final point, Beth says:

> Well, the problem is that the kids who have been learning in this very traditional school system want to continue to learn that way, and that's what they're comfortable with, that's what their background is.

Beth illustrates the superficiality of the adage about teachers teaching as they were taught. Despite a conventional and mechanical exposure to mathematics in her own education, Beth's disposition continually causes her to seek other models for how to teach mathematics and to appreciate those who taught her by example how to see mathematics as problem solving rather than as rote learning of operations. With one exception, the curricula of her college courses and the manner in which they were taught reinforce the findings and recommendations of the *Mathematical Education of Teachers Report* (CBMS, 2001a): educators and mathematics professors need to work together to create and teach courses appropriate to educating prospective teachers.

Beth's story also illustrates the need for continuing and consistent support for new teachers learning to teach mathematics. Her mathematics subject matter knowledge is no substitute for an interactive faculty, which is lacking in her current teaching position. Her story also illustrates the benefits of a consistent philosophy of mathematics education within a school. At Beth's school, the simultaneous use of two different curricula based upon very different philosophies, both within the school and within her classroom, contributed to student resistance to her using new curricula and new approaches to teaching mathematics.

Beth has all the intellectual qualifications and motivation to be a successful teacher of mathematics. Beth's story shows that talent, ability and a will to teach differently can overcome a lackluster K-12 education, that exemplary teachers can show the way, and that continuing education in mathematics could help further the process of a teacher's development.

George

> As far as opening my eyes a little bit, I think that the one-week TERC seminar opened my eyes to seeing that there are a lot of different ways of looking at a problem, and that counteracted the way I had learned mathematics.

George is an example of someone whose experience with mathematics in grades K-16 is very different from his experiences in his teacher preparation program and while teaching. He describes his early experiences with words like "tedious" and "boring", and he recalls all students "being taught one algorithm". By contrast, he describes his methods course and a professional development seminar, with words such as "playful" and "revelation", and he notes his excitement of seeing "different ways" to look at problems.

George, age 43, grew up in the 1960s and 70s. He describes the school setting as "traditional big classroom public schooling" where students sat at desks in rows, sometimes alphabetically. He describes the style of instruction he associates with school:

> I had pretty traditional schooling, you know, both in math and in reading. I mean in reading, we had Dick and Jane and Spot and Puff and 30 kids and you waited for 29 other kids to read their sentence in the room until your turn came along, and then you read your sentence.

He went to college directly after high school, majored in economics, and completed all but one course needed for graduation. He worked as a carpenter for fifteen years and didn't bother to take the remaining course until he felt that he had a reason and motivation to do so. That motivation came when, as a parent of a learning-disabled child, he became interested in education. He took the remaining course, in 1993, and immediately began the teacher education program. He soon discovered that teaching was a good fit for him. He describes himself as social, active, and creative, all qualities that he considers assets in teaching.

With the exception of geometry, George found school mathematics boring, tedious and uninteresting. He found it easy to do the mathematics problems he was given, but he disliked doing them. He describes his mathematics classes in elementary school through undergraduate work:

> You had one algorithm you were taught and you just got sheets of [math problems] … It was just going through steps, you know, you were just - it was just like a ritual type thing where you did this and this and this and this.

He recalls paying no attention to the work in his ninth grade algebra class for the entire year. He reports that, knowing he need only pass the New York State Regents Exam to pass the course, he opened the book and taught

himself the year's course material in two weeks time and got a 96 out of 100 on the exam. He took only algebra and geometry in high school, the minimum two courses required to graduate. He says he "got away with just one semester of college algebra". He took statistics for his major in economics and particularly enjoyed graphing and "mathematical like work and concepts."

High school geometry was the only K-16 math course he liked. In that class he got to do activities, such as bisecting angles with a compass. He finds it no coincidence that he is attracted to both geometry and carpentry. He says, "I think one of my strengths is being able to conceive of spatial problems in my head, which is very useful as a carpenter." He reports that he continually uses the skills he learned in geometry in his carpentry work. He also says that he uses mathematical reasoning and applies mathematical skills with facility when running his carpentry business.

Learning mathematics in a formal setting once again became an enjoyable experience for George when he took a math methods course in his graduate teacher education program. There he again did activities, got to solve "puzzles" and describes doing mathematics in class as something "playful." He reports that what was most

important to him about the class was "the added element of thinking about how kids are understanding math". It was the first time that he began "thinking about the thought processes that you're looking to build in kids and the types of activities that you can do, that it was useful. I enjoyed the math class."

While George identifies the math methods course as a significant turning point in his attitude towards teaching elementary mathematics, he expressed that it was only a beginning in his education. Of the course he says:

> I didn't feel like I had gotten nearly enough from coursework, from one – again, one semester, one course, covered all of elementary ed [math]. And there wasn't just enough sense of that practical well, where do you start? I need to – you know, I need to have a year's worth of stuff, so where am I going to get it?

He also speaks of the limitations of taking courses in isolation from teaching and of his belief that a single semester of student teaching gives pre-service teachers too little classroom experience. George addresses deficiencies in his teacher education program when offering suggestions for improvements:

> Well, I think I'd build it much more on the model of the trades, of apprenticeship. I think it would be mostly in practical, in the classroom, and with course work on the outside … You know, in some ways, while teaching is completely different from building, and it shares the same thing of it being very experientially based. You know, you really – learning to be a good teacher

> is an assembly of knowing a thousand different things, just as building a house is knowing a thousand different sub-skills. And really, the way you learn those things is by doing them.

Upon completing his teacher education program, George created an apprenticeship of sorts for himself: he chose to work as an assistant teacher before seeking his own classroom. George says his motivation to get more experience was influenced in part by his assessment of his readiness and in part by his perspective as a parent that the students in the classroom deserve to have teachers with more experience than that provided in the teacher education program. He says that he was lucky enough to get an assistant teaching job at a good school with an environment conducive to learning. George feels fortunate that the income he earned in his half-time work as a cabinet maker made it possible for him to continue his education by working which children in a classroom while having additional mentoring from the classroom teacher.

While not being solely responsible for choosing or creating curricula, as an assistant teacher George worked directly with students throughout the day. He and the teacher split the class for mathematics and George taught math to the first graders. He used a variety of curricula the teacher had selected and was exposed to a variety of manipulative materials and resources for teachers. Unlike

his own memories of elementary mathematics as algorithms and worksheets, the mathematics instruction in the classrooms in which he was a student teacher and an assistant teacher centered on activities, open-ended problems, and discussions about multiple solutions.

After three years in this position, George took a teaching job in a combined first and second grade classroom at his current school, a private school with a "progressive educational ideology." To prepare to use *Investigations in Number, Data and Space,* the curricula used at the school, the school administrator asked George to attend a weeklong workshop the summer prior to his starting. The workshop was sponsored by the creators of *Investigations*: TERC, a mathematics, science, and technology research and development organization.

George identifies this workshop as being especially significant in his development as a teacher of mathematics. He refers to the seminar repeatedly in both interviews and calls it an "eye opener" more than once. The seminar changed his view of mathematics and his view of himself as a mathematics learner. He contrasts his experiences in grades K-16 with his experience in the workshop. He says:

> I had fun there [at the TERC workshop]. And, you know, it was an eye opener, again, I think because I grew up so much - as I said, where it was - you were just being taught steps that was

> so boring that suddenly here was this idea that you could do different things different ways. And I remember one in particular where we were supposed to – in one of the little workshops, we were supposed to order fractions. And, you know, it was enough along in the week of having that idea of flexibility that I realized I could use common numerators to figure out fractions instead of common denominators. And it was such a revelation that – "Oh, that works, too."

George's view that he could make his own mathematical discoveries enlarged his view of what was possible for the students in his class to do. As in the math methods course he took four years earlier, the TERC workshop helped him focus attention on how children learn math. George also identifies using the TERC curriculum and reading the accompanying teachers books as important pieces in his learning to teach math. He appreciates the teachers' books for the clarity of the background information and for having examples of kid's work and student answers. He describes how using the TERC curriculum with his students and seeing their discoveries excites him. He says:

> And, you know, it stuck out for me as that revelation of breaking loose of always doing things in one way and the idea that you can look at things in different ways. So I'm always fascinated with that with kids of how – I mean that's something I really like in the whole TERC framework is how you're encouraging kids both to use a lot of different ways, to discover their own ways, and then to explain them. You know, last year I had a kid who had worked out a – his own algorithm for subtraction that required borrowing, and I'm not even sure I could remember exactly how it is, but it was completely

> different than anything I had ever thought of, but it works, and it works every time. And I was so blown away when he first explained it to me.

George values all that he learns from students and from teaching. He considers this learning from students to be a work in progress. He comments:

> And then there are all the things you learn everyday in the classroom from the kids and what is the variations in how they think about things and what it is that they are capable of grappling with…. I think that even after all the years of teaching math, I feel very much still in the learning phase – especially in how kids think of math and what they are capable of and all the variations. I am still learning a lot.

All of these experiences (his math methods class, his experience as an assistant teacher, the TERC seminar, using the TERC curricula, having access to the support material in the teachers books, being sensitive and listening to the students in his class) have contributed to George's learning to teach mathematics differently than he was taught. He does not tell or show students the "one way" to do things, but he values their discoveries. Students in his classes do the activities in *Investigations* in groups or as individuals, and George encourages them to find as many ways to solve the problems as they can. They then come together to discuss their solutions. He believes, "They have to explain how they solve the problem because that helps them clarify their own process and methods."

George not only appreciates the variations in how students approach math problems, he is sensitive to variations in how students approach learning. He continually strives "to better be able to individualize so that I can feel like I'm keeping the kids who need to be challenged challenged, and I'm keeping the kids who are really struggling with the basic concepts to feel like they're making progress and being successful."

In summary, except for geometry, George was turned off to school mathematics as a student. Outside of school, George reasoned mathematically and applied mathematics in his carpentry work and in running his business. Both his mathematics methods course and a professional development seminar gave him opportunities to focus on his own mathematical understandings and to think about how children think about mathematics. He discovered that he could have fun doing elementary mathematics and had strategies for making mathematics a fun and meaningful subject for his students.

Helen

> I have good memories of math when I was a student. I enjoyed it very much and I was confident with it. And then, when I became a teacher, it was really difficult for me because math came so easily to me as a student that it was hard for me to wrap my brain around the idea that kids learn in different ways.

The tension between Helen's own experience learning mathematics and the experience of her students is a major component in her self-perceptions as a mathematics teacher. The story of Helen is one of a teacher who has always considered herself a strong mathematics student. Unlike teachers who are insecure in their knowledge of mathematics, Helen was always comfortable with mathematics. Helen's story gives us the opportunity to examine the mathematics development of an elementary teacher for whom mathematics content is not a concern to her.

Helen started elementary school in 1970 and graduated from high school in 1984. In describing herself as a student of mathematics, and her feelings about mathematics, Helen uses only positive terms: "strong student," "did really well," "enjoyed it tremendously," "was confident with it," "really liked it," "loved it so much" and has "good memories". "I found every aspect of math to be fun. I liked to problem solve and figure things out up through calculus problems."

Helen believes the encouragement she got from teachers throughout her schooling was significant both in building her confidence and in her learning math. Helen stresses the importance of comments and encouragement by her fifth and seventh grade teachers and cites their encouragement as significant factors in her learning mathematics. "You think that side comments are insignificant in life but I think they really make a difference. We have to be careful what we say to kids."

Helen remembers studying "mostly numbers and operations" prior to high school, with geometry as a separate subject in seventh or eighth grade. With the exception of fifth grade, Helen does not recollect using many manipulatives or concrete materials. She says that her fifth grade teacher, who is now her colleague, was always "ahead of his time."

Starting in the fourth grade, Helen reports that she was always in the "highest track," taking her high school mathematics classes with the same 25 to 30 students, and that she was one of only ten students to take the senior math course in calculus. Helen says she responded to her first term calculus grade of D, her only academic grade below an A-, with determination and hard work, and "ended

up loving it [calculus] and bringing the grade up to an A by the end of the year."

Helen says she "breezed through" two semesters of college calculus "no problem", despite the fact that her high school "was not known for its academics." Helen also took a computer design course in college, which was somewhat mathematical.

After majoring in Interior Design and working for a few years after college, Helen says she ultimately decided to become a teacher because she "loved learning" and wanted to impart that love to her students and to "try to get kids excited about school like I was." She reports she was also influenced by having had some inspirational teachers, particularly her fifth grade teacher.

Helen entered the teacher education program in 1994. She remembers the math methods course as primarily teaching her "how to use manipulatives and how to bring manipulatives into the classroom" and remembers that the content included evaluating and organizing data. Helen does not remember the course specifically introducing her to the NCTM Standards, and said that she could not recollect ever having seen or read either the 1989 NCTM *Curriculum and Evaluation Standards* or the revised year 2000 *Standards.* Helen is, however, very familiar with the state framework

for mathematics and the learning standards for the students in Massachusetts and says refers to them often.

When speaking of the structure of the teacher education program, Helen says she would have liked more integration between the student teaching experience and the course work, and also a student teaching experience earlier in the program:

> I wish I had taken - I wish they made you do the student teaching at the beginning of the grad school experience, or in the middle or something, so that the next time I took a course, I would know what questions to ask and what kind of things to look for. Because it's very different. The reality is very different from the theory. It [student teaching] was a great experience.

Helen's confidence, knowledge, and comfort with mathematics were among the qualities and assets that led her to a student teaching placement with John, teacher of a combined fifth and sixth grade classroom at the Fifth Street School. Helen says she was attracted to the special math and science connection in that program. "I felt math and science were strengths of mine and I wanted to know what was the best way to teach them to kids so that they would be excited by them as well." As a cooperating practitioner training new teachers, John was particularly concerned that student teachers have a good command of all subject areas taught in the 5/6-grade class. Helen's

strengths in mathematics and science, and the fact that John's mathematics and science programs included the introduction of complex topics in these subjects, influenced both Helen and John in their mutually electing to work with one another.

During Helen's student teaching semester, mathematics was one of the first subjects John asked her to teach to a group of her own. For mathematics instruction, students were divided into two groups and John and Helen worked with the groups simultaneously. John further expressed his confidence that Helen could handle the subject matter by assigning her a group of seven students whom he considered to be the "advanced group." As a prelude to a unit on operations with fractions, John asked Helen to design and teach a series of lessons to help students understand the concepts of least common multiple (LCM) and greatest common factor (GCF). I observed her teach the first of these lessons in her third week of student teaching,

It is my experience that many elementary teachers are unfamiliar with the terms and concepts of LCM and GCF and would have found the task challenging. Helen, however, felt very comfortable with her assignment. She understood the terms and could procedurally find both the LCM and GCF. More importantly, Helen had played around with numbers, had

good number sense, and saw relationships between numbers. The lesson plans she wrote for the unit indicated that Helen did not plan to tell or show the students how to get the LCM or GCF and that she expected the students to make their own theories of how to find the LCM and GCF and to discover relationships between and among the numbers. For example, Helen states that students should be able to "test a theory that they [the students] have developed for finding LCMs, and refine that theory" (Lesson Plan, February, 1996). Helen explains that she wanted the students to see relationships between the numbers and to discover that one can find the least common multiple of a pair of numbers by dividing the product of the two original numbers by the greatest common factor.

Because Helen worked with the "advanced" group for most of the student teaching semester, she had little experience working with students who struggled with mathematics. Also, because she and John taught mathematics at the same time, she had less chance to observe John teaching mathematics than she had to observe John teaching subjects that were taught to the class as a whole. Helen's interest in using her early practical teaching experience to help her grow as a teacher was reflected in her

conviction that time observing other teachers is very valuable. She says:

> The best thing about student teaching was being able to just observe a teacher and see the kids' reactions and what they were interested in and what they got excited about.

Helen remembers one of John's science lessons as an example of student engagement in a lesson. In teaching about levers on simple machines, John had stood at the end of a board with a fulcrum and had invited the students to figure out how to lift him up. Helen describes the enthusiasm of the students as they discovered they could lift him up by adding weights to the other side and that the distance from the fulcrum affected how much weight they needed to add. Helen believes that the students will forever remember their solution to the problem and, hence, forever remember the scientific principles behind this demonstration. For Helen, the teachable moment was not just for the students but was significant for her as well. It taught her that active lessons grab students' attention. Helen is insistent that it is particularly important for teachers to have the time to observe one another. She said she "loves to go in and sit and watch other teachers, and how kids react to them, and [to] the body language of the teacher, the tone of the teacher." Helen believes that there are "so many things that you don't really learn in

the grad school course work. You don't really learn about that until you do it or see it." She also believes that it took doing a lesson and experiencing teaching in order to heighten her awareness of what to look for. She says, "I don't think I would have noticed as much about tone and body language until I actually tried to teach a lesson by myself."

After working as a teacher's aide for a year following her 1996 graduation, in 1997 Helen took her current position as a fifth grade teacher in the town's single middle school, a school for grades five through eight in the school district in which she had been a student. Some of her former teachers are now her colleagues, including Stuart, her inspirational former fifth grade teacher. She has a self-contained classroom and teaches all academic subjects. At least once a week, Helen and Stuart join their classes together for science and social studies. When asked to elaborate on a comment she made that it was hard to be an expert in so many subject areas, Helen stated clearly that she "likes the challenge", that she "enjoys teaching all subjects" and that she is glad that "she teaches elementary school". She referred to her positive experiences as a student and noted that she "did school well".

Helen says she avoided becoming overwhelmed her first years of teaching by focusing on only one subject area each year. "And, for the other subjects, I just used the textbooks and went through the motions with those subjects and hoped the kids would pick things up along the way." Helen says she started with language arts her first year because "reading, spelling, and writing together made up a good portion of the day".

She says she focused on social studies the second year because she was on the committee to rework the social studies curriculum. She reports that her work on social studies was important to her because she was "never a good social studies student" and that she is "learning so much about social studies now that I'm teaching it". She also says that when she was in school, social studies was limited to history which "was taught in a dry way", was not "real," and left her with no memory of what she had learned. She says she "wanted to do something different [in teaching social studies] for my kids." She says she has now broadened her definition of social studies beyond history and has become much more interested in "social change, social justice, and current events from a multi-cultural perspective than when I was a student." Today she

ranks social studies, together with mathematics, as her two favorite subjects to teach.

Helen says that it was not until her third year of teaching that she began to focus on teaching mathematics. It is noteworthy that Helen intermingles comments about learning to teach social studies with answers to questions about teaching mathematics. Her early focus on other subjects seems to be offered as an explanation as to why she hasn't worked more on her mathematics teaching. Also, Helen's statements indicate that over time she has developed a more positive attitude toward social studies and is learning by teaching it. This seems to stand in contrast to the continuing frustration she expresses at not reaching all students in math. "It still is hard going, trying to find other ways to explain something or, you know, other methods to teach [math]." She repeatedly expresses that her ease at learning math makes it difficult for her to think of ways to help students who don't "see it the way that seems so clear to me." She says:

> I think it was difficult for me to teach math to all levels of learners because it is so clear to me. Multi-step processes have always been very clear to me and it is hard for me to adjust my thinking to trying to meet the thinking of children who have trouble with multi-step processing and visuals. Starting to use manipulatives has helped me. I started using them last year. The first three years I was not a very good math teacher. I just repeated what I

> said for kids who didn't get it. Last year I started to try different ways to reach kids. When I had more time and it was less overwhelming.

Helen treats her mathematics teaching as a work in progress. She believes that she has "so much to learn and so much improving to do" in her mathematics teaching. When asked how she prepares to teach a new mathematics unit, Helen says she has been teaching "by the book" and skims to see what materials she needs to bring to class. She says she does not do the problems in the book before she assigns them to the students and is "sometimes caught off-guard because I hadn't prepared them ahead of time." In general, she criticizes herself for not spending more time preparing her lessons:

> I'm kind of lazy so I don't – I mean I don't do all that much prep, as much as I should. I hopefully improve year to year.

Helen does, however, refer to some improvements and changes she made to her mathematics teaching starting in the third year. She contrasts her students' reactions to studying mathematics before and after she introduced the use of manipulatives and project-based activities:

> I had more exciting hands-on projects for them to do and I noticed them [the students] getting more excited. Kids are very honest. During the first two years I noticed that, when I said we were going to do math, the kids kind of moaned. Last year when I said we were going to do math, they

> started to show more enthusiasm for the subject in general. They worked in groups. It was almost like they were helping each other more. Every aspect was improved - having them collect their own data instead of looking at charts in the book. It was more personal to them.

Even after making these changes, Helen says that she stuck to the book. She says that, one day a week, she introduces a new topic and the lesson is centered on a discussion and a mathematics activity. Such was the format of a lesson I observed her teach in May 2001, as she introduced a unit on geometry. Helen describes a lesson more typical of lessons she uses on the other four days:

> Typically I have them look in their book. We open to the page and they introduce a concept in the book and have sort of a fun drawing that helps them understand the concept. And then we work through a couple of problems together. And in the book they have a section called a guided practice, and then I let them work on the independent practice, and at the end of the class we go over the problems and I get the kids to help each other. And if I pull out one wrong, two wrong, three wrong - if a lot of people get four or five wrong, I ask them if they want a review, you know, that kind of thing. "Is there anything you want me to go over?" And then I go over it and then there's usually, at the bottom of the page, like a little project they can do….

The mathematics lessons Helen likes to teach are ones that have projects and "real world" links for the students.

In July 2001, Helen expressed her desire to move away from the book-driven lessons described above to more project-based lessons. At the first interview Helen she

said she was looking forward to using a new series adopted by her school district and hoped that the new series would be more "project based." In the follow-up phone interview with Helen four months after she had begun using the new series, Helen said that she liked the new series and that it was "much better for the kids." She describes the new series as having much more problem-solving, using hands-on and number sense activities to develop skills in estimation, incorporating discussion into the lessons, and including multiple activities for each topic. She likes the use of group work and the fact that she can see who is "getting it and who is not." She also feels that there is good support for the teacher and that multiple teacher guides give information about the mathematics and explain the authors' approach to content and pedagogy.

A new superintendent's commitment to the new mathematics curriculum also had two plusses: the district bought the accompanying manipulatives and hired the publisher to lead a series of professional development workshops for the staff. Helen says she found the professional development days helpful, particularly a demonstration lesson, which all the fifth grade teachers got to observe and discuss. Helen says that she welcomed the professional development in mathematics. Her other

professional development workshops, all offered by her school district, centered on other topics. Helen says the school had participated in a mathematics and science initiative. While she recalls some meetings to talk about science, she recalls little time spent on mathematics, and says that if there was time spent on mathematics it must have been when she was out on maternity leave. Even at monthly half-day workshops, when teachers present lessons and share student work with other teachers at their grade level, Helen recalls that mathematics lessons were rarely discussed. Hence, mathematics teaching, until recently, was not reinforced for Helen by in-service or other activities.

In conclusion there are several things to note when one looks at Helen's story. She often refers to the importance of her early experiences of learning mathematics in influencing her attitude about the subject. She has continued to learn about teaching in her early years as a teacher and prefers practical demonstrations. Helen also realizes there are other issues besides one's comfort and confidence with math that affect one's ability to teach it. She says she has not been very inner-directed in her choice of pedagogical training since her teacher preparation program; external events, such as a new textbook and new

curricular material, have caused her to react positively. The tasks of a beginning teacher might seem overwhelming and Helen's survival strategy has been to focus on one subject at a time, with the choice of which subject being partly circumstantial.

Because Helen feels confident in mathematics, she does not feel pressed to prepare as carefully for her lessons as does someone who might be less confident. Her comfort with mathematics, therefore, allowed her to take for granted her knowledge of math and may have been a barrier in her teaching to students who did not excel in mathematics. Finally, it is important to see that mathematics teaching development takes place within the context of teacher development in general and some teachers, such as Helen, might for one reason or another, give priority to developing their teaching subject areas over mathematics.

Jan

> And I'd never thought about - really thought about - math in a kind of meta-cognitive way. I just did it. I knew it and I did it. I never thought about - like - why these theories, or why this equation, or making sense of it. I just did it. And in Connie's [math methods] class, you had to think about those things because you had to think about how are you going to explain this to kids? And if kids can explain it or understand exactly what's going on, then it all makes sense.

Jan, a woman in her early 30s, graduated college in 1992, earned a MA degree in higher education and worked at universities for 5 years before entering the teacher education program 1997. After graduating in 1999, she spent the next two years teaching second grade in two different public schools in a large urban school district. She describes the school at which she taught the first year as "very, very traditional," where the principal talked with her more about changing her bulletin boards than about what the students were learning. She describes the second school, where she currently teaches, as one "closer to my student teaching experience… [There is] a great deal of collaboration, a great deal of creativity. Teachers are working together… So it's much more in line with the philosophy that I'm comfortable with."

Jan singles out three experiences she finds to be most significant in her learning mathematics and learning how to teach mathematics: struggling with math in her tenth grade

geometry course; taking Connie Brown's math methods class where she learned to view mathematics education differently than when she was a student; and using the TERC curriculum with her second grade students during the past year. Her own learning and teaching of mathematics illuminate why these three experiences were particularly significant to her.

Jan grew up in the 1970s and 1980s in a very large urban city. After going to a private elementary school, she went to an academic public high school to which students were competitively admitted by exam. She was an A student and math was one of her favorite subjects. She views herself as a math learner who "always liked math in school," and says that she "liked math because [she] was good at it" and because "it was logical and made sense." She describes the emphasis of her school mathematics instruction as one of "memorizing" formulas and procedures for doing problems without requiring students to understand why the procedures worked or where the formulas came from.

Jan took four years of math in high school: algebra, geometry, trigonometry and pre-calculus, and reports that the only math class she found difficult was geometry. She says that she managed to pass geometry only after working with a tutor and passing the New York State Regents exam.

She says that several years later she realized she was "terrible at spatial relations" and that her inability to see angles and triangles within a figure kept her from applying what she knew. She says:

> I knew all the theorems, I swear if I could figure it out, I could do a proof to death once somebody showed me look, this angle's in here and this and this, but I couldn't - visually, I couldn't make that make sense for me. I couldn't see it as I looked, and just saw all of these shapes on a page.

She says that her experience with geometry eroded her confidence and changed her view of herself as a mathematics student:

> [A]t that point, which was 10^{th} grade, I realized that maybe math is not this thing that I'm so great at.

She "loved trigonometry" and says that when she took trigonometry "[it] was back to normal math once again, and I could do it and it was fun. And it was challenging for me, which was really important."

When she got to college, Jan took college algebra and "hated it". She says the instructor gave little support and took the attitude that students were to "go do it [the assignments] and come back." Having fulfilled her one semester math requirement and finding that math was "no longer fun", she decided to take no more math courses in college.

Jan says that when she got to Dr. Brown's math methods course she discovered that "math had changed". She contrasts her view of learning math as a student in the 1970s and 1980s with her view of mathematics as she would teach it in the 1990s. She says:

> [When I was a student] we'd get these problem sheets and you just did it. But now, all of a sudden, in the class, I learned that there were all these tasks the kids are getting to do so that they understand the whole concept behind whatever it is that they're learning, which was very new. Very, very new for me, and I think very difficult.

In Connie Brown's class, Jan began to "explore all the whys" of math and to "question everything." Jan relates that she took away from the methods course a new view of mathematics:

> What was ingrained in me [by taking the course] is the idea of thinking about math differently, about how I teach it, how I learned, and how the kids learn about math. And so that's the piece that stays with me from Connie's class, and thinking about myself as a learner and putting myself once again back into the kids' place and really having them understand the concept.

Jan took the math methods course concurrently with her student teaching. Although both Jill, her cooperating teacher, and Connie emphasized the need for teachers and students to understand mathematical concepts, Jan found herself having to work hard to keep from teaching mathematics as rote memorization of operations. She was

also insecure about being able to teach math in a conceptual way. She says:

> …[I was] coming to it (math) as a teacher in a whole new age of education. … I always found myself kind of reverting back to the way that I did it. And now all of a sudden I had to relate it to the kids' lives; I had to make it real for the kids…. I was just thinking, "Oh, my God, I'm never going to be able to do this." I knew a simple way to do it [math]: memorize it, figure it out, and tell the kids to do it. But education had changed and math had changed, and so I was really, really scared about teaching math because it wasn't the math that I knew anymore. I was fighting with what you knew and what you learned.

In her student teaching class Jan used some of the "manipulatives" she learned about in Connie's class to accompany the curriculum they used in the class, *Everyday Mathematics*. While Jan now considers this curriculum "much more traditional than TERC," she finds it is still geared more toward developing student understanding of the math than were the texts she used as a student or the textbook series she was given to use in her first year of teaching in a very traditional school setting.

Jan's first year of experience teaching math is set against the backdrop of the school and context in which she taught. She says the tone of the school was set by the principal, who cared that the students were "quiet in the hallways" and "looked busy" and who did not seem to care whether "in fact [the students] were learning a lot."

Coming to observe her teach "only when it was time to do [her] evaluation", the principal "never looked at my kids' portfolios, never talked to the kids about their work" and focused his attention on checking for charts in which he mandated teachers keep records of such things as homework and compliance with wearing uniforms. The mentor to whom she was assigned taught by using worksheets and offered to share her worksheet collection with Jan. Jan says this "was not terribly helpful" as she did not use worksheets and preferred that her students spend their time reading and writing. Jan was discouraged, by the other teachers at the school, from trying new things and says that suggestions she made were met with comments such as, "Nah, that's never going to work: we tried that before."

Her math program the first year centered on the traditional workbook used throughout the school. While she did not particularly like the texts, Jan says she found it helpful to have the security of something familiar and easy to teach. She says that it gave her "something to run with" during that first year when the demands of learning to teach were too great for her to work on developing her teaching in all curricular areas at once. Nevertheless, Jan says she supplemented the text-based program with different activities, that she taught math in groups, that

she encouraged students to find different solutions to problems, and that she required students to discuss their solutions with one another. Jan says that, in her first year, she focused her attention mostly on literacy for two reasons: 1) she is passionate about reading and believes, "if you can't do that, [read], I think there's very little else that you can do"; and 2) she was concurrently completing literacy courses leading to certification as a teacher of reading.

Feeling "isolated" and "miserable" in her position, Jan looked for a new school and took a job teaching in a wonderfully supportive environment, a math and science pilot school for grades K-8 in which teachers work in teams. Jan reports two major factors that led to her focusing her attention on teaching mathematics her second year teaching: 1) using the TERC curriculum with her students; and 2) having a mathematics coach and other supportive teachers and administrators at the school with whom to talk about using TERC and about teaching mathematics. She says, "TERC forced me to think about math again, and what I think about it and how I teach it."

Jan explains that the structure of the school is conducive to interactions among staff members and to continuing professional development and growth for all

teachers. The school is K-8 and is divided into a lower (K-3) and an upper (4-8) school. Jan's team, consisting of the two second-grade teachers and two third grade teachers, meets with the coordinator of the lower school for two two-hour meetings weekly, once as a team and once with the other lower school staff as well. At these meetings they talk about curricular issues, subject matter and teaching, and Jan says teachers bring up specific situations in their classrooms and seek advice from one another. Jan says it is especially wonderful to "bounce ideas off other people and to bounce ideas off people who care." Each team member also specializes and acts as a resource in one of the four subject areas; mathematics, literacy, science and social studies (Jan's specialty is literacy). She says she learns from her colleagues and appreciates not having the burden of having to be a specialist in every subject area. Furthermore, the team also works together to develop the year's science and social studies units; all four teachers teach the same topic, adapting the level to their individual classes.

Jan says that changing school environments was critical in her development as a teacher in general, and specifically in her development as a teacher of mathematics. The new environment provided support for

teaching a more challenging, and ultimately more satisfying and effective, curriculum in mathematics. She says that it is important to her that she is no longer in a "sink or swim" situation, but that she gets support from her team members, her coach, and the administrators at her school:

> The expectations are different. The focus is on student achievement. … I needed to be in a place where the focus was on teaching so that I could get good at it. I need to be around people who know teaching and who do teaching, who love teaching, and who can help me improve my practice. And that wasn't going to happen there [at the first school]. And that was very obvious. And so - it happens here on many, many different levels.

Jan says she was ready to tackle TERC in her new school because she knew that she would have support from her colleagues, her administrators, and a coach who would observe her teaching weekly and would discuss her lessons with her. Unable to make it to the summer TERC training, Jan prepared to teach the curriculum by working through the exercises herself and going through the teachers' manuals with the other teachers at the school. After her first year of teaching TERC, Jan has mostly accolades and only a few reservations about the curriculum. Most exciting to her was that she learned math by teaching TERC and that she learned both *with* and *from* her students. She says:

> And the kids here have been doing TERC for three years at this point; they've been doing it since K-1. And some of the things cycle through, like

> they have this game called "Guess my Rule" that they all know and they love and it's wonderful. And they have several different things like that. So they're teaching me at the exact same time [they're learning].

Jan says that with TERC, "math is fun again", for both her and her students, and that everyone looks forward to math time. She finds that the curriculum is "easily differentiated" and that it is easy to both supplement activities and challenge students. She appreciates the fact that students do not all have to be doing the same thing at the same time; TERC recognizes that students develop their mathematical understanding at their own pace. She finds the teachers' manuals explicit and appreciates the information they give and their inclusion examples of student solutions to problems.

Jan says, however, that she was nervous about teaching the geometry unit and confided in Lilly, her math coach, relating the story of her dismal high school geometry experience. Lilly assured Jan that she would like the way TERC presented geometry. Jan says that not only was the geometry unit "lots of fun and the kids had a blast with it", but teaching geometry ultimately changed Jan's view of the subject and her view of herself as a math learner. She says:

> If maybe I'd learned some of it this way, it would have made sense to me later on. We didn't

> do any geometry in school when I was a kid. I don't remember anything aside from when I was in Kindergarten and learning shapes. … Some of the things that these kids were doing, I mean they were doing halves and wholes and thirds and they were just doing things that I know I didn't do in second grade.

Jan says that her ability to converse with Lilly and with Matt, the other second grade teacher at the school, about the curriculum was invaluable to her. She says that Lilly influenced her development as a math teacher in several ways. Coming into the room each week, Lilly served as another pair of eyes. Not only did she have advice for Jan, she was able to make suggestions on how to help individual students. Lilly often worked with a group or an individual student, thus relieving Jan of these responsibilities. Lilly was the coach for all of the second and third grade teachers. In the follow-up conferences, Lilly met sometimes with Jan alone, sometimes with Jan and Matt, and sometimes with the whole team. Lilly was able to refer to ideas she had seen in other classes, and when there was a concept with which students in several classes were having trouble, the teachers and Lilly brainstormed together. Jan also appreciates the fact that Lilly has been working in the school for some time and has built a relationship with the students. Lilly was coaching Jan the day I was observing Jan teach. On Jan's suggestion, Lilly

worked individually with a couple of students so that they would have extra and more challenging work.

Jan has concerns about the TERC curriculum. She finds it very language-based. She has several students who are still not proficient at reading and finds that some of the language is difficult for even the good readers to understand. Jan says that she is also concerned that she is unable to do all of the books at her grade level. She asks, "If we've only gotten through four books and there are seven of them, well, what happens to those three other units? Is it picked up later?"

The lesson of Jan's story is that learning is a social experience. For Jan, the environment at the second school at which she worked has made a tremendous difference to her growth and development as a teacher of mathematics. Jan's story also shows the importance that curricular materials play not just in students' learning but also in teachers' learning. Jan says that during her first year of teaching she had more confidence in her ability to teach reading than she had in her ability to teach mathematics. She says for her first year of teaching she was comfortable using a mathematics curriculum similar to the one she had used when she was an elementary school student. She says that starting her second year of teaching with confidence in her

ability to manage a classroom and with the knowledge that she had supportive colleagues, allowed her to feel secure and comfortable about trying a new mathematics curriculum. As she used this curriculum, she found it helped her to develop her mathematical understanding. Jan is an example of a teacher who continues to learn from other teachers, from using rich curricula, and from her students.

Rita

> As a math student, I struggled from day one. I remember tears constantly over math programs. I did not like it. I hated it. You know, in arithmetic, the sheets, endless sheets of papers and worksheets of just filling out, counting the apples; that's what I remember, you know, the basic equations and the flashcards. I mastered the flashcards and I remember feeling very proud of my multiplication tables and division, and then pre-calculus, I did that and I moved along. I was steady paced and in advanced classes in math all the way through high school.

Rita, who is in her early thirties, currently teaches kindergarten in a suburban public school. Rita recalls very little that is positive about her school mathematics experience. She recalls getting "straight As" in math, which she says she got because she "studied hard" and that she "never really understood what I was studying". As a middle school student she was placed in the highest level of tracked mathematics classes, algebra in middle school and pre-calculus in the ninth grade. She recalls that she did not continue in the track that led to calculus and that by sophomore year, she was no longer "in the higher classes". She gives no details of her three years of high school math courses saying only that: she was one of "very few girls" taking pre-calculus; she took geometry in tenth grade; she can't really remember her junior year course and she did not take mathematics her senior year. She sums up

her high school math experience: “I struggled the whole way; it was painful.”

She recalls that, as a freshman in college, after she “failed” a placement test, she “ended up taking pre-calculus again; the same class I took my freshman year [of high school]”. She says she is “embarrassed” to say that she failed the college pre-calculus course and had to repeat it. She describes the second experience:

> It was a huge lecture hall. We went in, I couldn’t understand the professor. And then I took it [college pre-calculus] again. I got a C- or something. [After that] I took a statistics class, a business statistics class, accounting classes. I could not do math. I mean I remember - it was just awful.

After college, where she majored in English, she got a paralegal certificate. She worked as a paralegal for five years, including two in the real-estate department where her job largely consisted of calculations. She reports that she was “stressed out” by doing math and that she was “not confident” in her calculations.

Rita identifies three experiences as significant in her learning mathematics and learning to teach mathematics: her math methods course with Connie Brown, her student teaching experience with Nancy Hall, and her team teaching experience with Paula White.

In the first of these three experiences, the methods class, Rita reports having a "rebirth of sorts" and a "new perspective" on mathematics. Rita says that for the first time she had fun with math and that she discovered relationships among numbers and operations that she had never thought about before and that she was very excited about her new understandings in a variety of mathematical topics. She says: "[E]very class I went to, I just went off in another direction in terms of 'Like wow, I get it.'"

In her description of the class and its lasting impressions on her, Rita says that she appreciated both the content she learned and the context in which she learned it. She attributes the comfort with which she felt she could explore mathematics to the tone in the class set by Professor Connie Brown. She describes Connie as the "neatest woman in the way she approached everything" and as a woman who brought laughter to the class and who allowed students enough time to explore mathematics. She says that in class Connie talked about giving enough time to children to explore mathematics and that she practiced that philosophy on the adults in the class as well. Rita particularly appreciated that Connie the emphasized mathematical reasoning and importance of explaining one's

answers and not merely getting the answers themselves.

Rita says,

> I absolutely loved it [math methods course} because it was the first time I understood what - 2 plus 2 plus 2 was, like what it *really* was… -. I felt like a child - I would say, [for] about 95% of the time in that class - doing things for the first time… Connie made it okay to not know the answer to things and, [she was] just wanting you to reason. Like she always wanted the reason why you got to something and I think that needs to be stressed more. And I'll always remember that, I think, - instead of like, "What's the answer?"

Rita says that the mathematics methods class allowed her an opportunity to explore her own mathematics using concrete materials and "manipulatives" that Connie introduced in class. Rita recalls that the only concrete material she used in her own education was a wooden clock on which she could move the hands when learning to read time. Rita explains that using these materials in Connie's class helped her explore elementary mathematics, introduced her to materials she could use with children, and helped build her confidence in teaching mathematics. Rita says that in class she learned that there were different ways of approaching and solving a problem. She says that for the first time math was "logical", "it made sense." She says that she "felt dumb that I didn't know it before… [that] there's other ways of solving things."

Tina's own mathematics education is, in many ways, a work in progress. It was impeded by her own limited education as an elementary and high school student, by math methods instruction that ignored the conceptual difficulties in mathematics in favor of less important techniques, and by work situations where she was expected to use new and challenging curricular materials without adequate preparation. She was aided by an outstanding mentor teacher who helped her face and overcome her own conceptual limitations, and ultimately by a supportive school environment. Tina still has not received sufficient formal instruction in the TERC materials. She shows limitations originating in her own schooling in specific subjects like geometry. Yet along the way Tina's own self-awareness and the fortunate experiences that (to her credit) she exploited now put her in a position to continue to develop herself as a mathematics teacher.

Chapter V Cross-Case Analysis

The previous chapter consisted of roughly chronological versions of the experiences of seven teachers, showing their development as math teachers to this point in their careers. In this chapter I consider these experiences in a more aggregated form. What was common and what was different in the teachers' experiences? What were the similarities and differences in the backgrounds and early education of the teachers? Expressed in the words of the seven teachers, what influenced their mathematics education? I seek to evaluate the significance of their experiences, drawing primarily from the accounts of the teachers themselves but augmented by my own observations and notes. The narratives were analyzed from three perspectives: the educational contexts of pre-college and college education of the seven teachers, the types of experiences that were critical in their learning to teach mathematics, and their current experience as teachers. I first summarize the findings and then present and analyze the results.

Summary of Themes

From the interviews and observations the following findings emerged:

1) All seven teachers report that their early experiences with mathematics in school greatly influenced their view of mathematics as a discipline and their view of themselves as learners of mathematics. All seven teachers describe their elementary and secondary mathematics education as "traditional", i.e. math learning centered around worksheets in which one applied rules, procedures, algorithms that their teachers had demonstrated.
2) Six of the seven participants tell of experiences in which they revisited their own mathematical understandings during their pre-service education. They say these experiences allowed them to view mathematics as a discipline differently than the way in which they were taught mathematics and allowed them to reassess their view of what elementary mathematics was and how deeply they understood it. From the perspective of their years of teaching thus far, they all saw tremendous value in this rethinking of elementary arithmetic and geometry as adults. They talk about four categories of significant experiences through which they revisited their own mathematical understandings:

A) Teachers and mentors: significant people who facilitated their learning mathematics and learning to teach mathematics

B) Curricula they used for teaching, which served as a vehicle for their own learning, by challenging their own conceptual understanding, and as a vehicle for the mathematical development of their students

C) Professional development workshops, coaching and other experiences

D) The culture and environment of the school and administrative leadership

3) The seventh participant, Helen, stands apart from the other teachers in two ways: 1) she was more content than the other teachers with her mathematics instruction as a student and 2) she does not relate revisiting her mathematical understandings during her pre-service education or student teaching experiences. Helen does, however, speak about revisiting social studies as a discipline, and she talks about her development as a teacher of social studies in many of the same terms the other teachers use to describe their changing perspectives of mathematics education. Like the other teachers, Helen identifies the role of

mentors, curricula and professional development as significant in her development as a teacher of mathematics, but she does not speak of these experiences in terms of revisiting her mathematical understanding.

4) All seven participants consider their growth and development as teachers of mathematics to be works in progress. In particular they do not separate their development as mathematics learners and teachers into three periods: schooling (where they learned math); teacher education (where they learned the techniques of teaching); and teaching (where they learned in a practical manner how to apply these earlier forms of education). Their learning mathematics and learning how to teach mathematics are seen as intertwined with one another during all stages of their careers. The teachers also say they learn how to teach from the students in their classes and through the act of teaching.

I now discuss these themes in greater detail.

1) Early experiences influenced the teachers' views of themselves as mathematics learners. All of the teachers were themselves taught mathematics in a traditional setting using algorithm-based curricula.

How the participants' describe their school mathematics experiences matches the findings of many

researchers who found that prior to the mathematics reforms of the 1990s, elementary school math was primarily limited to numbers and operations and that elementary and secondary mathematics instruction was centered on teachers showing algorithms to students.

Despite the fact that the high school graduation dates of the participants spanned a fourteen year period, 1975 to 1989, participants' descriptions of the environments in which they had studied mathematics in their K-12 schools were strikingly similar to one another. All seven participants described the schools they went to and the type of mathematics education they got in elementary and secondary school as "traditional". To them "traditional" education is one in which their teachers lectured and demonstrated a single procedure or algorithm for a particular type of arithmetic problem; students then applied this formal procedure to similar problems. Sara noted that her "K-12 math was pretty traditional. Nothing stands out for me except sitting behind a desk and looking at a blackboard." George considered this type of mathematics instruction "a ritual type thing" while Rita remembered "the sheets, endless sheets of papers… counting the [pictures of] apples." Only one participant, Helen, recalled having a teacher who routinely used concrete

materials in teaching mathematics. Three other participants recalled the use of a single concrete or manipulative material, (namely a wooden clock, wooden numbers and symbols, and Cuisenaire rods) by a single teacher in a single year of elementary school.

The high school math classes of all the participants were taught lecture style. All participants reported that they did well in elementary school mathematics as taught in this traditional manner and six reported that their elementary work and test scores led to their placement in "high track" middle school classes or, in Jan's case, admittance to a public high school which competitively selected students on the basis of grades and exam scores. Hence, the sample defined itself as a group of very good elementary school math students, who were all taught in a traditional manner.

Nevertheless, their grades and performance in elementary school mathematics did not always translate into self-confidence in their mathematical abilities, as evidenced by Rita's statements.

> As a math student, I struggled from day one. I remember tears constantly over math programs… I was steady paced and in advanced classes in math all the way through high school.

The participants had different reactions to their traditional elementary mathematics programs and had mixed

experiences with middle and high school mathematics. They developed different concepts of themselves as math students. Their self-assessments were sometimes based on their teachers' stated opinions of their mathematical abilities. In some cases they seem to have arrived at their self-assessment independently of their teachers.

For example, George and Beth based their views of themselves as math learners on experiences outside their classes. George reports doing well and feeling that math was easy in school, but disliking the way math was taught. He said he actively decided not to pay attention in his algebra class, ultimately teaching himself algebra and doing very well on the New York State Regents exam. He reports that he loved geometry class and later used the skills he learned in that class in his daily work as a carpenter and small business owner. Beth said she did well in school; ending up in the highest track in math classes, but later failed high school calculus when she did not work in the class. However, scoring the highest grade in a high school mathematics competition, which tested problem solving ability, as well as taking a course in polyhedral geometry in a college program for high school students, restored her self-esteem in mathematics. She emerged with a positive view of mathematics as a discipline and of

herself as a math learner. Interestingly, both of Beth's affirming experiences valued building conceptual understandings of mathematics and not solely computational proficiency.

Five participants viewed themselves largely through their performance in school. Of these only Helen had an entirely confidence-building experience. She says that she liked school, liked studying mathematics in school, did consistently well in mathematics, and felt that she was supported and encouraged by her elementary and secondary mathematics teachers. She considers comments made by teachers with regards to her mathematical ability important to her self-concept. Helen says:

> I remember him [her seventh grade teacher] telling me that I had a propensity to do math and that encouraged me. You think that side comments are insignificant in life and I think they really make a difference. We have to be careful what we say to kids.

She says that she has always liked to play around with numbers and she easily sees relationships among them. She says she overcame the only bump in her mathematics studies, a low first term grade in senior year calculus, by conscientiously doing the work and ultimately loving the course and getting an A in it. Thus for Helen, her positive experiences in school reinforced her positive view of mathematics and of herself as a math learner.

Tina, Jan, Sara, and Rita each had their confidence in their mathematics ability shaken by specific experiences with school mathematics. All four said that they were good math students in elementary school. While high grades and test scores in elementary school mathematics qualified them to be placed in the high-level math classes, each had at least one confidence-eroding experience in secondary school mathematics, which colored their view of themselves.

Of the four, Tina and Rita report never feeling that they were strong math students and stated that they did well only by studying, working hard, and memorizing facts and procedures. Tina says:

> I think math for me was a subject that was a little bit puzzling. I just - it didn't come easy. It's something I felt like I had to memorize in order to understand it. And pretty much, that's how I survived.

While Sara and Jan report they like math and were initially confident in their mathematical ability, they, like Tina and Rita, report loosing confidence in their ability to learn math once they hit subjects in which they did poorly in school. Jan says:

> I always liked math as a kid, and the reason I liked it is because I was good at it… Until I got to geometry, which I could not figure out. I could not figure it out. It just didn't make the logical sense of everything else…But it was at that point, which was 10th grade -when I realized that maybe math is not this thing that I'm so great at.

Tina, Rita, and Sara report that their difficulties with a math course resulted in their "demotion" to a lower track of math classes, sapping even more confidence. Sara's statement is representative:

> But then when I went to high school, only the really super, super smart kids stayed in Level I [classes], and the strong kids and the typical kids were in Level II. So that was a little bit of a little letdown, like all of a sudden I went to Level II and I felt really average. So that was a little bit discouraging.

The teachers report that their high school experiences with math often influenced their college course choices. Helen says calculus was a "breeze' because of the strength of her high school course. Beth and Tina both use the word "stubborn" to describe their motivation to take calculus their first semester of college: principally to prove to themselves that they could do it. Tina did poorly in calculus and Beth became a math major. Rita reports feeling embarrassed at having to repeat her high school pre-calculus course and managing to fulfill the remainder of her four-semester college mathematics requirement with statistics and business math courses. George, finding college algebra no more engaging than high school algebra, and Jan, finding college algebra class lacking in the kinds of support she had gotten in high school math class, both took a single semester of math, the minimum college

requirement. Sara says she managed to avoid taking mathematics at all in college.

This experience of initial proficiency followed by discouragement and abandonment of the study of mathematics is a very general phenomenon. The National Research Council report, *Everybody Counts* (National Academy Press, 1989) addresses this very issue.

> More than any other subject, mathematics filters students out of programs leading to scientific and professional careers. From high school through graduate school, the half-life of students in the mathematics pipeline is about one year…Mathematics is the worst curricular villain in driving students to failure in school (Everybody Counts, 1989, p.58).

Referring to educational practice that "offers mathematics students only a dim light at the end of a very long tunnel", the report argues, "we need even more to increase illumination in the interior of the tunnel" (Everybody Counts, 1989). Several of the teachers in the study indicate that later in their studies, light was offered by key individuals.

None of the seven participants report taking a mathematics course in college that included studying the mathematical concepts or the underpinnings of the content of elementary mathematics. The teachers report that they learned numbers and operations in elementary school and algebra and geometry in high school. Jan says, "Like

that's where I learned all the math that I'm teaching now – I learned it in elementary school."

Beth feels similarly about where she learned the mathematics she now teaches in the middle school.

> I mean I was a math major. But most of what I teach I learned in elementary school, middle school, maybe high school, and then was reintroduced maybe in Connie's [math methods] class.

2) Opportunities to revisit their own mathematical understandings were significant to the teachers' growth and development as mathematics teachers.

Dissatisfaction with their own elementary and secondary school mathematics instruction was a motivating factor in the teachers' desire to find new modes of learning and teaching mathematics. With the exception of Helen, the participants in this study say they were inspired to make the mathematical experiences of their students different from their own.

The six teachers talk about having revisited their own mathematical learning as adults, and, as a result, having come to view elementary mathematics as more than algorithms, procedures to memorize, and a search for "the one right answer". The six teachers talk about discovering that there are several ways to solve any one mathematics problem. This is what George calls the "revelation of breaking loose of always doing things in one way."

They each tried to replace the view of elementary school mathematics as rote memorization of operations with numbers with a view of mathematics as problem solving, relationships, communication and connections. These moments of insight came at different times in their careers and for each person there was a different stimulus and a different opportunity. In most cases, however, the teachers report multiple experiences through which they gradually changed their conception of math. For these teachers there were in general four different sources of growth and change in learning to teach mathematics: inspirational teachers, mentors or colleagues, challenging curricula, professional development activities, and administrative leaders and school environments conducive to learning.

A) Learning to Teach Mathematics with the Help of Influential and Inspirational Teachers and Colleagues:

The teachers most often cite their teachers, their mentors, and their colleagues as most significant in their learning to teach mathematics. These transforming encounters occurred at different points in the teachers' education and careers. For most of the teachers these encounters stimulated a relearning of mathematics. They relate they "learned" the math they teach in their own elementary or secondary school experiences, but they say

they "relearned" mathematics and learned how to teach mathematics almost exclusively during their graduate teacher education program and from other teachers while teaching.

Beth was the only person to cite a college professor as influential in her revisiting her own mathematics and in her learning to teach mathematics. She names Dr. Janet Jones, her advisor and abstract algebra professor, as the one professor within her college mathematics major who taught math the way she wants to teach math: by challenging students to think and apply their knowledge to new situations. Beth says Dr. Jones' class inspired her to work hard:

> I just worked so hard in her class. I think her exams were more an extension of what we learned rather than a test of what we learned. We had to - like I remember she gave us - a lot of the exams were take home exams. So the problems would be harder or different. So we'd really have to think about it rather than just solve them.

Beth also says that Dr. Jones "taught the way the way that women learn", which she describes as "nurturing", fostering discussion rather than competition among the students in the class, and creating an environment in which students asked questions for understanding and not to show off their knowledge to the professor. Beth says that she is "a very visual person" and that she appreciated Dr.

Jones's use of visualization in the abstract algebra course, something she found missing from her calculus courses. These more innovative approaches were experienced in higher-level mathematics but Beth had no difficulty in seeing these as approaches that could work at any level. Her appreciation of Dr. Jones, was mostly retrospective:

> Now that I am in the teaching field, I analyze things from a teacher's perspective. And then [when I was in college] I didn't so much… But once I got to [the teacher education program], I really started observing the teachers teaching and I started observing the way I learned. And I think in a way the process of doing that was in some ways more valuable than what I was learning, though what I was learning was important, too.

Beth says that Connie Brown, her math methods instructor, reminds her of Janet Jones in that she got the students to think about the mathematics. Rita and Jan, both of whom also had Connie Brown as their math methods professor, found their first opportunity to revisit their mathematical understandings in Connie's class. Jan and Rita both say that Connie pushed them to think about the processes they use to solve mathematical problems and shifted the focus away from the answers. They described Connie's class as one in which they engaged in mathematical activities and problem solving in several areas of mathematics, not just numbers and operations. They both say that in Connie's class they learned that what was

important was understanding the concepts, being able to reason mathematically and being able to communicate to others how they solved a problem. Rita also appreciated the lightness and humor that Connie brought to the study of mathematics and the respect that Connie showed for the math methods students as learners of mathematics themselves. She says that Connie spoke of children's needs for varying amounts of time and experience to understand a concept in math, something Rita understood in terms of her own experience. Rita says:

> [T]he adults that were taking the [math methods] class with me would get it in a second, and I was still focusing through – I mean working through- things in my mind - how to solve equations. So I think it was neat to see that kids might have these same issues as I did as an adult.

Rita says, "Every class I went to, I just went off in another direction in terms of wow, 'I get it.'" The course also put Jan in the position of a learner and this caused her to "think about math differently." Jan and Rita both say that engaging in problem solving in Connie's class allowed them to relearn the mathematics. Rita says for the first time she "understood what 2 plus 2 plus 2 was…what it really was." She mentions one class session, taught by a practicing middle school teacher, on multiplication and division of fractions as being particularly enlightening to her.

In the methods class, the study of the math itself was intertwined with the study of how to teach it. The five teachers who cited the importance of their math methods course indicated forcefully their appreciation of the abilities of their professors to bridge the gap between doing mathematics and teaching mathematics. The participants report that through doing mathematical activities in class: ***they*** learned the math. They thought and talked about how children learn math, and learned strategies for teaching children math. The emphasis was on process and on understanding math as they thought about how they could help students understand math. Teaching mathematics provided an incentive to understand it themselves. As Jan said, "In Connie's class, you had to think about those things because you had to think about how are you going to explain this to kids. And if kids can explain it or understand exactly what's going on, then it all makes sense."

Sara and George, who both had Wilma for their mathematics course, also talk about the bridge that Wilma, as a practicing teacher, built between philosophies of teaching mathematics and what works with students. Sara says that one way she revisited the teaching of mathematics was in the field experience that Wilma designed to

accompany the course, which involved observing and/or working in a variety of elementary school math classes.

Two teachers who did not find the math methods class to be particularly significant in their development were Helen and Tina. Helen says her math methods class taught her about using manipulatives, but that the course did not really focus on content. Tina felt her instructor gave assignments which did not increase her knowledge of either mathematics or how to teach mathematics, and which did not challenge her to learn geometry, the mathematical subject she felt she most needed to learn.

The teachers who found their math methods course to be significant in their learning to teach mathematics talked about learning both content and pedagogy and about their concern about knowing what they will need to know to teach.

> At [college]… I just want to get the grade so I can graduate…but at [my teacher education program] it was more, "I really want to learn this." You know, it was a very different experience. The grade so much wasn't what was important; it was absorbing the information and figuring things out.

The teachers in this study also discussed the deficiencies in these courses on teaching mathematics. Their primary complaint was that the courses were not long enough to allow an investigation of all of the topics they needed to study.

Another potential pivotal figure in the re-learning of mathematics for teaching is the cooperating practitioner. All but George name their cooperating practitioner during the student teaching experience as a particularly influential person in their learning to teach. Four of the teachers, (Sara, Tina, Rita, and Beth) single out their cooperating practitioners as particularly influential in their learning to teach mathematics. The teachers served as role models through their ability to bring theory of math education into real life situations and as examples of successful teachers who revealed their struggle with their own mathematic understandings.

For example, Tina and Rita, who both had Nancy Hall as a cooperating practitioner, say they found Nancy's life experiences particularly instructive. Nancy told them that she had had difficulty in learning math. Her current mastery inspired and reassured Tina and Rita. It occurred to Rita that Nancy might have become such an effective teacher of mathematics in part because of her struggles. Rita says:

> Watching [Nancy] behind the scenes, though, and knowing the work she puts in to understand [math]. 'Cause she doesn't get it either, sometimes - like she does get it but she - I remember her like working through it, you know? Like not just pulling sheets out. She was working through to solve things. I think that's neat to see.

In addition to mathematical activities at a set point in the day, Tina and Rita say Nancy's math program was integrated into the day; there were math activities and applications in everything the students studied. Rita and Tina attested to Nancy's very clear sense of the scope and the sequence of the concepts. She always developed a conceptual understanding, through stories and activities. Nancy extended the concept of the single math learner to the concept of the class as math learners. It was particularly important for these apprentice teachers to witness an entire class developing together an understanding of a mathematical concept, followed by activities in which the students could apply their new knowledge.

The impact of exemplary lessons coupled with Nancy's humor and sympathetic personal story of her difficulties with math were a very potent influence. Attesting to this influence, both Rita and Tina gave examples from their own teaching, where they now use activities they had first learned in Nancy's class. The level of mathematics that the first and second graders in Nancy's class could do, the level of complexity they could understand and use, surprised and impressed Tina and Rita. They both say they saw math in a new way. They feel they were students

themselves, and were learning along with the first and second graders.

For all of the student teachers, teaching was a qualitatively different experience from their previous math education. They learned to think about what it was the children knew already and what were the conceptual ideas they needed to know before going on to the next step. With their cooperating practitioners, education in a classroom setting also became highly individualized. They talked about how individual students learn and they confronted how they learn. They also talked about learning about practical issues of teaching math and about the organization of the classroom for teaching math both during the math hour and at other times during the day. They report talking with their cooperating practitioners about the preparations for teaching, including the physical preparations they need to make before each lesson and the practical strategies for moving the lesson along. Four teachers, Sara, Beth, Rita, and Tina, say their mentor teachers were particularly influential in their learning to teach mathematics. All four teachers give examples of things they did or learned from their cooperating teachers and give examples of specific approaches they now use in their own classrooms.

All seven teachers say they continued to find mentors and supportive teachers who significantly facilitated their continuing development as teachers during their first or second teaching positions. Though there were differences in their experiences, overall I would say from my experience as a supervisor, this group was fortunate. Sara, George, and Rita, worked as instructional aides upon graduation and speak about the added value of working closely with another teacher and having the opportunity to have yet another perspective on teaching math. Sara speaks about Maureen, a fourth grade teacher for whom she was an instructional aide:

> It was [Maureen's] eighth or ninth year teaching and she shared a lot with me about her first couple of years teaching… And so I learned a lot about how to be a teacher from her, you know, the things they don't really teach you that much in school. [For math] she had a lot of lessons start with the kids writing in their journals in response to an open-ended question.

Although not officially assigned as mentors, both Jan and Beth say the teachers on their teaching teams acted as mentors. While she did not particularly have math support from her co-teachers, Beth had a "great" principal and much support as a new teacher at the school of her first teaching position. Beth says:

> I didn't have official support but I felt very supported by my team. Because everybody else had been teaching for 20 years and they were so

> great. I mean, they were so happy to have somebody new, and it was great. So even though there wasn't, you know, a mentoring program, I felt taken care of.

B) Learning to Teach Mathematics: the Value of Challenging Curricula

A second factor influencing both their understanding of math and their development of means of teaching math was the curricula the teachers used in their classrooms. All seven teachers made significant efforts to teach the newer curricula based on understanding and problem solving. The teachers say these new curricula challenged their own conceptual understanding of the material. They were often forced to seek support to answer student questions. They sometimes had difficulty anticipating the kinds of issues the students would stumble over. For any given topic, the various curricula might be adequate or inadequate in giving them the background information they felt they needed to make their own conceptual transition. Though this was stressful, it was also a growing experience for the teachers.

Circumstances and curricula were of critical importance in encouraging the teachers to avoid the traditional ways of math instruction. For example, Jan and Sara each started out at schools where they used workbooks, which reminded them of the books they had used as students.

They both say they supplemented the workbooks with other activities, but that was not the same as using a more challenging curriculum. Sara and Jan say that when they took jobs at other schools and had a chance to work in settings in which they used curricula that were activity-based and modeled on the NCTM standards, they had a chance to learn from the curricula.

The importance of the curriculum as a device for teacher learning is clear from Jan's experiences. Jan makes several comments about how using an activities-based curriculum provides opportunities for students and the teachers to construct their own understanding of mathematical concepts. She says:

> And TERC is something that you can learn along with the kids… clearly you've got these big math concepts like yeah, I know how to add, I know how to multiply, but you're learning along with the kids at the same time so that's exciting and that's new 'cause I'm learning the curriculum and how to teach this curriculum and how to present it.

Jan distinguishes between two types of mathematical concepts she encounters when using this curriculum: 1) concepts, such as the concepts behind arithmetic computations, she feels she "knows" but needs to rethink in order to teach conceptually and differently from the algorithmic way she learned and 2) mathematical topics, such as geometry, that she did not study in elementary

school and does not feel she understands. She speaks about her own mathematical understandings in these two areas and her learning the math by teaching the curricula in these areas.

Jan says she rethinks concepts she understands when she prepares to teach the curricula by working through the exercises herself. She says:

> I know the concept [behind addition of two digit numbers with regrouping], I know I learned it in elementary school, but I have to go back and remember the concept of stuff that's automatic to me now so I can begin to teach it to the kids in a very different way… So I start first by going through it and then asking clarifying questions for myself, and then saying, "Okay, what are the things that I need to make sure that I do with the kids?" Because I know there are steps that I'm going to miss because in my head, I get it. And because it's so different for me, I need to learn it. And so I need to teach it to myself first. And so I teach it to myself first… I'll go through the lesson. I literally will go through the lesson. And as I teach it to myself, I'm thinking about my whole range of kids.

Jan says sometimes she learns from the curricula by trying to understand the reason particular activities are included in the curricula. In those instances, she says she feels fortunate that she has colleagues with whom to discuss the curricula. She says:

> I didn't get this whole pulling out the 10's. I was like - What are we doing this for? I don't get it. Why are they pulling out the 10's? Why don't we just do the algorithm? And then [my teammate] kind of explained it. I ask a lot of questions. Like if there's something that comes

> up in TERC that I don't get why we're doing it this particular way, I understand what the outcome is but why are we doing it this way, I ask.

Jan says she learns from the curricula when thinking about how she will teach it and by talking about strategies for teaching it with colleagues and with students:

> I'm thinking about, "Okay, you know what? On this part, [a student's] going to get stuck here. Why is she going to get stuck and what am I going to do? How am I going to break this down, and how can I make this simple?" And I don't always have those answers. And so sometimes I go back, you know, to my team or whatever. Sometimes I ask the kids, you know, kids who I know that will get this and say, you know, "I need to ask this question. I was thinking about doing it this way." Because they get it in their own heads and they're like, "Oh, why don't you just do it this way," and I'm like, "Thank you."

Hence, Jan says she learns math from teaching the curriculum, by working through problems, by probing for a deeper understanding, and by asking questions of herself, her colleagues and her students. She says she also learns math from teaching the curriculum, by the questions it stimulates in her students and by the mathematical explorations built into the curriculum:

> They [the students] also ask a whole slew of questions that I'm sure wouldn't come out in [a] kind of traditional math [program]. Like I know [questions] didn't come out last year when I was doing [workbooks in a traditional program], and [they] come out this year. And there's a whole lot of "what if." And you know what? [I say] "Let's try it. Let's try it. Let's go for it." There's room for that. Exploration is a good

> thing in this particular program. So I'm comfortable with it and I'm comfortable with the idea that I don't have to know all the answers and we can find them together, and I know that we can find them in the math curriculum.

Jan says that when it was time to teach geometry, she told her math coach about her own experience with high school geometry and expressed her negative feelings about the subject. She reports being impressed that her students could understand topics, such as symmetry, that she did not remember studying in elementary school. She says, "If maybe I learned [geometry] this way [in elementary school] it would have made sense to me later on. We didn't do any geometry in school as a kid. I don't remember anything until I hit geometry [in high school]."

Not all teachers say that they were able to learn mathematics from the curricula. Tina says generally that in the past, the teachers' guides have sometimes assumed that she knew material that she never learned herself. Curricula often introduce concepts that were in fact gaps in the teacher's own knowledge. How these omissions and gaps are perceived depends on the background of the teachers. For example, Both Jan and George were enthusiastic about using a challenging curricula and found the curricula to be a source of their learning math and how to teach it. Tina admitted that she sometimes had difficulty in seeing the

big picture. She also had problems with unfamiliar terms and new mathematical concepts. By contrast, George and Jan welcomed the new curriculum. They were aided by a formal introduction to the curriculum from an in-service workshop and/or from insights of other people familiar with the curricula. The learning experiences for the teachers, like all other learning experiences depended a great deal on the context, in this case the support from other teachers and the availability of other professional activities.

In the follow-up conversations, Rita, Tina, and Helen say they have been using a new curriculum this year and that they have had curriculum-specific professional development workshops to go with the new curriculum.

C) Learning Mathematics through Professional Development

A third major opportunity to revisit their mathematical understanding for the teachers was through professional development. The teachers spoke of five different types of professional development activities in which they were engaged: single day (or half-day) workshops, in-class mentoring or coaching, summer institutes, semester-long courses or multi-session seminars, and regular opportunities to meet with other teachers and curriculum specialists to discuss educational and curricular issues. Nearly all of the teachers engaged

in professional development experiences at their job sites. They chose topics that supported what was emphasized and valued at the school. Often, there were mandated professional development activities for the entire school staff and the teachers were constrained to focus on specific educational issues (such as learning disabilities), specific themes (such as teaching about diversity), specific subject areas (such as literacy), or specific pedagogical issues (such as classroom management or organization). The teachers say that math was rarely the primary area of focus. Even on the complete list of possible professional development classes and workshops offered by their schools or districts, the teachers report that there were few and sometimes no courses or workshops specific to mathematics.

All but Sara, however, had managed to engage is some professional development activity specific to math. However, the teachers report that many of the professional development activities involved minimal engagement and did not contribute much to their learning to teach mathematics. For example, Rita's school had a workshop on using a manipulative material that her school ultimately decided not to purchase. Beth took the only math-specific workshop her first school district offered – two half-day sessions

on strategies for teaching fractions. While Beth found some of the strategies useful, she says the workshops were conducted lecture-style and the instructor left without even allowing time for questions.

There was some value to conventional workshops. Rita and Helen took multi-session workshops this year in conjunction with the introduction of mathematics curricula their schools had recently adopted. At Helen's school, the workshops included a demonstration lesson to a fifth grade class with an opportunity for the teachers to discuss teaching strategies. Helen says that she found the demonstrations particularly helpful. Both Rita and Helen say that a combination of a school-wide curriculum (with support materials) and an opportunity for the teachers to talk about the curricula seemed most useful.

The two teachers who report the most extensive professional development were Jan and George, both of whom taught at schools which had adopted, TERC *Investigations in Number, Data, and Space,* a curriculum that approaches problems from individual perspectives. Both schools also created an environment conducive to using such a curriculum and both schools paid for summer institutes to introduce the teachers to the curriculum and the philosophy of the program. I will discuss Jan's experience in the next

section, where I also consider the school environment. Professional development allowed George to revisit his own mathematical understanding in a weeklong intensive summer workshop given by TERC. George said this was particularly important for his development as a mathematics teacher. He expressed experiencing the same joy at making mathematical discoveries as did his students during the year. George reports:

> "In one of the little workshops, we were supposed to order fractions… Having that idea of flexibility … I realized I could use common numerators to figure out fractions instead of common denominators…It was such a revelation that – "Oh, that works, too."

D) The Influence of School Culture and Environment on Learning to Teach Mathematics

The school environment is where the teachers first experience the limitations of their previous education and face their own personal challenges in becoming teachers. Not surprisingly, several of the teachers identify the school environment as important in their further development as math teachers. All the participants in the study taught at more than one school and almost all compared the school cultures of their two schools and the effects the cultures had on their learning to teach. In each case the principal set the tone for the school environment. In determining what was valued at the school,

the principal influences the nature of the math education of the teachers

For example, the principal at Jan's first school talked with her only about bulletin boards and keeping kids quiet in the hallways, and never talked about educational issues. By contrast, her current principal gives top priority to curricular and educational issues. Also, the second principal has built a community in which teachers share in decision-making about these issues. Jan says she now works in a team with the four second-grade and third-grade teachers. The team members interact daily and meet twice a week with the lower school principal to talk about curricular issues, teaching strategies, programs and the overall goals of the school. One of these two weekly meetings is with the kindergarten and first grade teachers as well. In this environment she feels supported and this facilitates her ability to learn from her colleagues and administrators. Jan articulates several ways in which school-wide programs support her development as a mathematics teacher: 1) As discussed in the curriculum section of this analysis, the school has adopted a curriculum that Jan finds allows her to grow as a mathematics teacher 2) The school pays for teachers to go to a one week summer institute for teaching math in the

constructivist way promoted by the curriculum. Because Jan was unable to attend the summer institute, the principal and experienced teachers met with over the summer to review the curriculum with her and to introduce her to the philosophy and goals of the program 3) Time for grade level team meetings is built into the school schedule, facilitating Jan's interactions with people who have experience in using the curriculum; these people are a source of help in both the mathematics content and pedagogy. 4) The school has hired a math coach for each team. Jan's coach, Lilly, comes weekly and observes teachers, models lessons, works with groups of students, and discusses mathematics teaching with Jan individually and with the whole team. Jan says that Lilly is a big support to her in both content and pedagogy and that Lilly serves a valuable role on the team in several ways: she oversees the math program, she coordinates the scope and sequence of the units, she provides articulation between the second and third grades, and she coordinates assessment efforts for the program. Jan felt that all of these features of the environment at the second school were conducive to her learning and development as a teacher.

The teachers in making choices about the schools at which they worked were also making choices about their math

education as one component in the school environment. Since the school environment was often a major factor determining which teaching positions they took, in effect the teachers were choosing their math curriculum and support community in choosing their school. For example, after a year of teaching at her first school, where she was "pretty much miserable all year," Jan had taken the initiative to seek out a school with an environment more conducive to learning - both the students' learning and her own learning. She recollects:

> I needed to be in a place where the focus was on teaching so that I could get good at it, you know? I need to be around people who know teaching, who do teaching, who love teaching, and who can help me to improve my practice. And that wasn't going to happen there. And that was very obvious. And so - it happens here on many, many different levels.

George stayed in his assistant teaching position until he found the right school environment for his job. George says:

> I didn't want to just be teaching anywhere for the sake of having the job, and I was in a really good school and I think I waited to get an opportunity that I felt was another really good school.

When George did find that school, he found the environment very supportive to his development as a math teacher. Not only had the school adopted a challenging curriculum, but also it paid for him to attend a summer

professional development workshop in which he revisited his own mathematical understandings.

Not all teachers could choose their school environment. Sometimes teachers took the only job they were offered. Sometimes administrative snafus on the district level sadly led teachers to leave positions at which they would have otherwise stayed. Such was the case for Beth, who left a wonderful inner city school. It had a "great principal" who was a former mathematics teacher, a supportive team of mentor teachers, a school environment which built community spirit and respect in a racially, ethnically, and economically diverse community, and an atmosphere that supported her teaching a challenging curriculum in mathematics. Beth finds that in her second position in a suburban, middle class, "traditional school system" she has less support for trying new curricular materials. Many of the students and teachers at the school are resistant to modernizing their teaching of mathematics. Opportunities for growth in her mathematical content and pedagogical knowledge have been more limited in this new environment than they were in her first school.

In the follow-up interview, however, Beth reported that this year her second school has a new principal who is beginning to institute changes at the school. Beth says

that the principal is persistent despite meeting some resistance among teachers who have been at the school a long time. Beth says, "There is a sense that 'we have always done it that way.'" She is finding that the principal can effect changes in areas such as curriculum selection; yet, teachers are the ones ultimately responsible for implementing those changes.

What emerged from the teachers' stories is that both the principal and the greater culture and environment at the school are important factors in teacher-learning. The principal sets the tone, but who the teachers are and how they interact with one another as a community is also key.

The following features of the school environment emerged from the interviews, as what teachers think is most conducive to their development as teachers of mathematics:

1) A principal who values mathematics instruction that is inquiry-based and who is supportive of teachers changing their practice to teach mathematics that way
2) Opportunities for faculty to interact and communicate regularly about curricular issues in general and mathematics curricula in particular
3) Adoption of interesting mathematics curricula rather than workbooks with a focus on computation

4) Frequent discussions and meetings of the teaching staff with administrators and/or mathematics curricula specialists to talk about educational issues

5) A principal and staff who value experimentation and trying new things

6) Opportunities to have professional development specific to mathematics

7) Accessibility of people who are knowledgeable in mathematics teaching in the new curricula

8) Investment in supplies and equipment that support the curricula (manipulatives)

3) When the teacher does not consider her mathematics knowledge to be an issue: Helen as a teacher whose energies are initially concentrated in other curricular areas.

Helen stands apart from the other teachers in the study in that she was more content with her early mathematics education than were the other teachers. She is the only teacher who named an elementary school teacher and a middle school teacher as having been significant in her development as a mathematics learner and teacher. She positively recalled her fifth grade teacher, a man she describes as being "ahead of his time" as introducing activity-based problems to study math. Helen is also the only teacher who did not relate an experience during her

pre-service education in which she revisited either her own mathematical understanding or her views of mathematics education. While the other teachers say they first confronted their mathematical understandings or their views of how best to teach children mathematics in either their mathematics methods class or their student teaching semester, Helen recounts neither her methods class nor her student teaching experience as particularly pivotal experiences in her development as a math teacher. In Helen's narrative she paints a uniformly positive view of her own mathematical understandings. Yet, she says this understanding still leaves gaps in her teaching, as she candidly admits that she has trouble teaching students who have difficulty with mathematics. I treat Helen's case as a special case because I believe as a teacher who does not initially focus on her mathematics understanding there is much we can learn from Helen.

As discussed in her case narrative, Helen reports only confidence-affirming experiences with mathematics as a student. Her school experiences in the highest track of mathematics classes from fourth grade through senior year calculus coupled with her student teaching experience working with the "advanced group" in math, gave her little preparation for working with students who struggle with

math. While she talks about her cooperating practitioner as a role model for teaching in a way that make ideas come alive for the students, she does not talk about learning to teach mathematics during that time. She did not work with all of the students in the class for mathematics and she does not talk about thinking about how children learn mathematics. She repeatedly contrasts the ease with which she learns mathematics to the difficulties she has teaching mathematics to students at different levels:

> I learn [math] from just reading the instructions or listening to the teacher, and no problem. But it amazed me that some kids couldn't do that and it was hard going. It still is hard going, trying to find other ways to explain something or, you know, other methods to teach it, So it's actually ends up being a struggle for me as a teacher, I think, more so maybe than others [teachers], because … it doesn't compute that they [the students] don't understand it the one way that just seems so clear to me.

Helen repeatedly affirms her confidence with the subject matter of mathematics. This implies that she does not feel as much urgency to learn the subject matter of mathematics as other teachers might feel. Her confidence with math also seems to affect how she approaches teaching mathematics. For example, the other six teachers talk about preparing to teach a math lesson by working through the student problems at home before assigning them. Rita's comment is typical:

> …Basically, I work through it [the math problem or investigation] myself. Am I giving someone - a child - something that I can't do or that I don't know how to approach? …[By] working through [the problem], I guess. I'm really preparing myself.

By contrast, Helen says: "I don't usually do [the math problems] before class. I usually do it along with them (the students). And sometimes I'm caught off guard because I hadn't prepared them (the problems) ahead of time. So that's something I should probably do, but -"

On the surface, it might seem that Helen is not as conscientious a teacher as the other teachers. A closer look, however, reveals that her approach to teaching and preparing to teach is not that different from the other teachers. It is just that her focus in her early teaching experience has not been on math; it has been principally in other areas. Helen explains that to "survive" the complexities of all there is to learn as a beginning teacher, she chose to focus on only one subject area each year. She says that she chose language arts her first year because it dominated the school curricula. She says she focused on social studies the second year because she was on a committee to revamp the social studies curriculum in the fifth grade. She says, "By the third year, I was ready to tackle math." While there was a somewhat external factor in Helen's choice to tackle social studies before

she tackled math, I think it noteworthy that in both interviews she mentions, unprompted, that she was not a good social studies student. I find Helen's view of social studies as a discipline and of herself as a social studies learner and teacher similar to the view some of the other teachers have with regards to mathematics. Helen says:

> I was never a great social studies student… but I'm learning so much about social studies now that I'm teaching it… as I get older I get more interested in this. I am interested in social change, social justice, and current events from a multi-cultural perspective - more now than when I was in school. When I was in school history was taught in a dry way. I don't remember anything I learned in any class - nothing felt real to me and I want to do something different for my kids.

In both interviews, Helen talks about her dissatisfaction with the way she had been teaching mathematics during her first years. She candidly says that she was not a very good math teacher for the first three years and that she "just repeated what she said for the kids who didn't get it." She says that it was only in the third year that she began to use some manipulatives and says that this has helped some of her students. Helen description of a math lesson that is typical of one she uses four days a week is reminiscent of Hiebert's (Hiebert, 1999) description of a common math lesson. Helen says she goes over homework, introduces a new type of problem, does a couple sample problems with the students, gives the

students practice problems in class and assigns homework. I think it significant, however, that the lesson she chose to do on the day I visited was nothing like the one she described. Instead, the lesson I saw, a geometry lesson to introduce a new unit, consisted of a thirty-minute discussion in which Helen asked the students what they thought geometry was while she recorded their responses on the board. Helen put the responses into categories as she recorded them. For example, she separately listed two-dimensional figures and three-dimensional figures. She did not, however, ask the students what classification she used for the categories or what was similar about the items in any one category. After the discussion, Helen gave the students prints of art work in which they looked for geometric shapes and then asked the students to make their own drawings using as many geometric shapes as they could. Helen said that after that lesson, she used the textbook to teach the unit on geometry.

In the follow-up interview this year, Helen describes her excitement and pleasure in using a new mathematics curriculum and in having had the support of materials and professional development to accompany her use of the new curriculum. She says the new series has a few workbooks rather than a textbook. It is activities-based and has

four or five investigations for each concept. It comes with a discussion book that fosters students talking about how they solve problems. Helen reports putting more energy into teaching mathematics and into looking over the new curriculum before she teaches lessons. She is pleased with the students' response to the materials and with the student's greater enthusiasm for studying mathematics. She says "this is better for the students" and that they are "less stressed out" and "getting what they need." Helen did not yet talk about the curriculum particularly challenging her own mathematical understanding.

4) Teachers say that their development as teachers of mathematics are works in progress

It is particularly interesting that almost all of the teachers consider their development of expertise in elementary mathematics to be ongoing, even as they improve their pedagogical methods. Some comments are:

> I think that even after all the years of teaching math, I feel very much still in the learning phase - especially in how kids think of math and what they are capable of and all the variations. I am still learning a lot. (George)

> Math is an area where, I think, as the years pass by I will get better at it. Time itself will help - more practice.... I actually like math and feel confident in my own problem solving and my own mathematical ability. I am new to teaching and am not confident that I am helping students problem solve... With every added experience I feel that much stronger. (Sara)

> Well that - I mean I still feel like I have so much to learn and so much improving to do. (Helen)

While the teachers all express a sincere interest in improving their mathematics teaching, they also relate that as relatively new teachers they have many demands and interests that compete with their efforts at improvement. This is the real world of teaching, where aspirations, and plans run into the issues of time and energy. These tradeoffs are particularly important in the willingness to continue professional development in mathematics. Tina, Rita, and Sara, name mathematics as a high priority area in which they would like to take a professional development workshop or seminar. However, all three of these teachers have new teaching jobs this year and say that the demands and requirements of their jobs often determine their professional development opportunities and choices for the year. For example, Tina's district required her to take two multi-week professional development courses, one for beginning teachers and one in science. She did not find mathematics on the list of other professional development activities, but plans to find a course or workshop to take next year when she is more settled in her job.

George, who found the professional development seminar he took in mathematics very valuable last summer plans to take a course in teaching beginning reading and in working

with students who have learning disabilities. He says that for now he feels that he is "in control of the math. Things are working well and [he] knows where the resources are", and that he needs to know more about how to teach beginning reading to his first and second graders.

The teachers talk about continuing learning to teach mathematics from other teachers. Tina finds her mentor a helpful resource and has been meeting regularly with her and with another teacher at her new school to plan and talk about their mathematics program. Beth has made plans to observe seventh grade mathematics teachers in other school districts where they are successfully teaching the curriculum she is using. She has also started to meet weekly with another mathematics teacher at her school who is enthusiastic about the curriculum to talk about mathematics teaching. Desiring to learn more about teaching students with learning disabilities, Beth took a college course in differentiated learning. She felt fortunate that the course instructor had degrees in mathematics and in education and that she drew the course examples from mathematics.

Helen says that this year the combination of a new and challenging mathematics curriculum, support materials to accompany the curriculum and school-wide professional

development around using the new curriculum has helped her to begin to teach mathematics differently. This year has been a turning point for Helen. She has gotten started teaching mathematics differently and is finding it to be an ongoing process.

Teachers in the Middle

The teachers in this study found themselves in the middle of school mathematics reform. They were elementary and secondary students before the 1989 publication of the NCTM *Standards*, and they became teachers after the curriculum reform movement had begun to change the way mathematics was taught in elementary school. Their experiences as elementary and secondary students, where they learned mathematics procedurally, contrasted sharply with their experiences as beginning teachers, where they were expected to implement mathematics instruction based on conceptual understanding.

These teachers related that as elementary and secondary students they applied procedures without understanding the mathematical concepts on which these procedures were based. They also said that their own elementary mathematics education did not include some mathematical topics, such as geometry or probability, which are now part of the elementary curricula. Reflecting on

their college math courses, from the perspective of being a classroom teacher, they noted that their college mathematics courses did not help them deepen their understanding of elementary mathematics. Since they felt they needed to understand elementary mathematics well in order to help their students develop their own conceptual understandings of mathematics, this was identified as a problem.

Six of the teachers in this study sought out experiences in which they actively engaged in learning elementary mathematics as adults. These experiences were situated in their teacher preparation courses, their student teaching experiences, and their teaching experiences and were facilitated by their interactions with inspirational teachers, mentors and colleagues; their use of challenging elementary school curricula; their participation in professional development workshops and seminars; and their interactions with supportive administrative leaders and their placement in conducive school environments.

Although I have characterized the concept of "being in the middle" as something that happened to this group of teachers at a particular historical moment in U.S education, I believe this phenomenon will continue to occur

over a rather long period of time. Educational reform in the U.S., and in particular mathematics educational reform, does not occur in a monolithic and synchronized way. Five of the seven teachers in this study recently spent at least some time teaching in elementary and middle schools where the predominant curricula and approaches to teaching mathematics were "traditional" ones. Hence, to the degree that there are schools and teachers today who are teaching elementary and secondary mathematics procedurally, for at least the next ten or twelve years we will continue to have people entering teacher education programs who were primarily educated in "traditional" ways. In addition different school districts adopt different curricula and different teachers will be trained in different ways. The population of these seven teachers actually encompasses a fourteen-year age span, and yet their mathematics experiences in elementary school were remarkably similar to one another. Therefore, I fear that the phenomenon of teachers finding themselves in the middle of mathematics reform will be with us for a very long time and will continue to pose a challenge for reform in mathematics education. The findings of this study and the implications of the findings will continue to be relevant for some time to come.

The findings of this study, however, are also extremely encouraging. The participants in this study were not recruited from the population of teachers who were in in-service programs and who had already chosen serious supplementary mathematics training. Yet, early in their teacher education programs and teaching careers, six of the seven teachers encountered people, situations, and experiences which stimulated them to re-examine their mathematical understanding and which contributed to their development of mathematical content and pedagogical proficiency. By the end of the study, the seventh teacher also was experiencing a similar effect stimulated by a new superintendent's commitment to support the implementation of newly adopted curricula with materials and professional development.

Hence, while being in the middle of mathematics education reform is a problem for teachers, the teachers in this study found ways to work on this problem through various means.

Chapter VI: Conclusions and Implications

The major conclusions of this study focus on the ways teachers learn to teach mathematics for understanding. They need to make a major transition from a view of mathematics instruction based on rote learning of mechanics to instruction based on inquiry and problem solving. There needs to be opportunities for each teacher to make this transition, which requires deepening their own mathematical understanding. The original question of the study, "How do teachers attempt to complement their mathematics learning?" was the main subject of the narratives about these teachers. While the path each teacher took was different, the overall conclusion about the importance of these opportunities to reconsider elementary mathematics was quite similar. The defining features of these opportunities were circumstances where the teachers could think as adults about the basis of elementary mathematics. Most often this rethinking was guided by practicing teachers and math educators and was stimulated by the use of new curricular materials.

An important insight from this study is that while the teachers sought different solutions, they reported similar effects on their teaching of the new approaches to math education. As they began to develop a new understanding of

what mathematics instruction could be, they also began to realize how critical their own mathematics knowledge was to their ability to teach mathematics in the way they wished to teach it. Given the motivation to better understand elementary mathematics, the teachers were receptive and resourceful in finding a variety of ways to educate themselves.

Though most educators believe that such rethinking should continue to occur at all stages of the teacher's development, the teachers' experiences emphasized the importance of having at least some opportunity to re-examine mathematics during the education of new teachers. Interestingly, most of the teachers in this study managed to find, or create for themselves, experiences that facilitated their rethinking. Yet, by their own accounts these teachers echoed the theme that there seems to be insufficient time in educational programs of teachers for such an explicit commitment to an exploration of the deeper content knowledge of mathematics. A major implication of this work then is that time needs to be set aside, in college or in teacher education programs and in in-service programs for new teachers to allow teachers and prospective teachers to re-engage in exploring the basis for mathematical concepts, including concepts underlying the

various areas of mathematics now taught in elementary school. A further implication is that those educators who are involved in the education of teachers need to be aware of the roles they can play in facilitating elementary teachers' development as teachers of mathematics. Drawing from the participants' experiences and the findings of this study I make the following recommendations for those involved in the mathematics education of elementary school teachers:

General recommendations for all groups

- Recognize that most current and prospective elementary teachers learned mathematics procedurally when they were elementary school students and do not have an adequate conceptual understanding of the mathematics now expected to be taught in the current elementary school curriculum.
- Recognize that many elementary teachers have had at least one negative experience with school mathematics when they were students and that this negative experience could have had a major effect on blocking them from furthering their mathematical education and in their ultimately becoming effective teachers of mathematics.
- Recognize that negative experiences with mathematics education as students leads some teachers to resolve

that they do not wish to teach mathematics the same way they were taught it. You might be able to help teachers appreciate the beauty of mathematics and to find other ways to teach mathematics.

- Realize that many teachers and prospective teachers are eager to learn more mathematics and welcome the chance to learn mathematics in a manner that will allow them to build their conceptual understanding.

College and University Mathematics Faculty

- Design and teach a course on problem solving and mathematical reasoning suitable for all university students.
- Structure your classes so that your college students engage in their own mathematical inquiry into elementary mathematics.
- Collaborate with faculty in schools of education and school districts on the design and implementation of mathematics courses appropriate for elementary school teachers. Spend some time observing elementary school mathematics lessons and pre-service mathematics education courses for teachers to get a fuller picture of the knowledge and understanding of mathematics that elementary teachers need. Co-teach some university courses with mathematics education faculty.

- Make yourself familiar with some of the inquiry-based elementary school curricula the teachers will be using to know the mathematical concepts teachers will need to understand.
- Read the *Mathematical Education of Teachers* [CBMS, 2001 #335; CBMS, 2001 #361] and use it as a resource guide for the mathematical content teachers need as you design mathematics courses for prospective teachers.

Pre-service Teacher Education Program Designers and Faculty

- Collaborate with mathematics department faculty and school district personnel to design programs for the mathematics education of pre-service teachers to insure that pre-service teachers re-examine the content of elementary mathematics sometime in their university or education.
- Broaden the curriculum of "math methods" courses to include examination of mathematics content and not just strategies and materials for teaching mathematics. Broaden the curriculum of courses in mathematics pedagogy to include the study of all topics in the elementary school mathematics curriculum (numbers and operations, algebra and functions,

geometry and measurement, data analysis, probability and statistics).

- Introduce pre-service teachers to a variety of inquiry-based mathematics curricula.
- Use an inquiry approach to teaching pre-service courses in mathematics pedagogy.
- Place student teachers in schools and classrooms in which teachers use an inquiry-based approach to mathematics teaching. For cooperating practitioners who wish to improve their mathematics teaching, offer joint in-service/pre-service workshops in mathematics teaching for the veteran teacher/student teacher pairs.

Co-operating Practitioners and Mentors/Colleagues

- Recognize that student teachers and novice teachers should be learning mathematics content along with how to teach mathematics. Discuss the conceptual understandings they will need with student teachers well in advance of assigning lessons they will teach and give them time and support for exploring the mathematics they will be teaching. Also it may be important to reassure new teachers, who have not had the opportunity to re-examine their mathematical understanding, that the process takes time and is worth the effort it takes.

- Make explicit to student teachers and novice teachers the thought processes you use in preparing and implementing mathematics lessons.
- Meet with other cooperating teacher/student teacher or mentor teacher/novice teacher pairs to talk about mathematics teaching or to attend professional development seminars in mathematics education together.

Principals

- Foster a climate of support for teachers learning mathematics and learning to teach mathematics in a manner that fosters conceptual understanding in students as you set the values, tone, and priorities for the school.
- Seek out opportunities to improve your own understanding of mathematics and mathematics instruction so that you can offer useful supervision to your teachers.
- Recognize that the mathematics curriculum used in the classroom often determines the extent to which teachers will use an inquiry approach to teaching mathematics.
- Recognize that teachers' prior experiences with mathematics might have left some gaps in their

knowledge that require them to have support using new curricula.

- Build in time in the weekly schedule for teachers to communicate with one another about teaching and curricular issues and designate some of that time to be spent on discussions about mathematics curricula and teaching mathematics.
- Offer professional development courses and workshops at the school that allow teachers to engage in their own investigations into mathematics. Find out from the teachers the areas of mathematics about which they would most like to learn.
- Forge an alliance with resource teachers, curriculum specialists, or math teachers at nearby high schools or colleges who could answer mathematical content questions teachers might have.
- Facilitate giving teachers opportunities to observe one another teaching mathematics.

School Districts

- Financially support administrators, teachers, or teams of teachers, in their taking summer workshops and institutes that allow them to re-examine their mathematical understanding.

- Support principals with funding for mathematics professional development activities for their staff and/or provide district-wide professional development.

Professional Development Providers

- Design courses and workshops to involve attendees in active participation and opportunities to engage in their own mathematical investigations.
- Survey potential participants as to what areas of mathematics they want to work on and design courses and workshops in those area. Focus more on content issues than on techniques and manipulatives.

Novice and Veteran Teachers

- Don't limit yourself to scheduled local professional development workshops in mathematics. There are some excellent mathematics institutes for which you might be able to get support to attend. Alternatively, if you do not have the offerings you need in your district, you might join forces with other teachers to lobby the district to bring in good mathematics professional development opportunities.

Curriculum Developers

- Survey teachers who use your curricula to see which mathematical topics might need additional clarification or background information in teacher support materials.

- Offer professional development institutes and workshops to introduce your materials and to allow teachers to re-examine the mathematical concepts embedded in the curricula.

In conclusion, this study points to the value, even the necessity, of an adult rethinking of the basic principles that underlie mathematics for those who plan to teach mathematics. Though the need for adult reflection is most acute when there are profound changes in educational curricula, there is always a need for such a re-examination as one is learning to teach. In fact, the one teacher in the study who did not initially re-examine her mathematics was limited in her ability to teach mathematics to all her students. My final recommendation is that all those involved in the education of elementary teachers take into account the importance of providing opportunities for adult re-examination of the basis for elementary mathematics. Those involved in the education of teachers need to first be convinced themselves of the need for and value of such a re-examination, and then need to work together to create experiences through which teachers will also be convinced of the benefits of such re-examination.

Bibliography

Adams, T. L. (1998). Prospective Elementary Teachers' Mathematics Subject Matter Knowledge: The Real Number System. *Action in Teacher Education, 20*(2), 38-45.

Ball, D. (1988). *The Subject Matter Preparation of Prospective Mathematics Teachers: Challenging the Myths*: National Center for Research on Teacher Education NCRTE-RR-88-3.

Ball, D. (1989). *The Subject Matter Preparation of Teachers. Issue Paper 89-4* : Office of Educational Research and Improvement (ED), Washington, DC. (EDD00036).

Ball, D. (1991). Research on Teaching Mathematics: Making Subject-Matter Knowledge Part of the Equation. In J. Brophy (Ed.), *Advances in Research on Teaching: Volume 2: Teacher's Knowledge of Subject Matter as it Relates to their Teaching Practice* (Vol. 2, pp. 364). Greenwich, Connecticut: JAI Press, Inc.

Ball, D. L., & Lampert, M. (1998). *Teaching, Multimedia, and Mathematics: Investigations of Real Practice*. New York: Teachers College, Columbia University.

Ball, D. L., & Wilson, S. M. (1990). *Knowing the Subject and Learning to Teach It: Examining Assumptions about Becoming a Mathematics Teacher* (Research Report 90-7). East Lansing, MI: National Center for Research on Teacher Education.

Borko, H., Eisenhart, M., Brown, C., Underhill, R. G., Jones, D., & Agard, P. (1992). Learning to Teach Hard Mathematic: Do Novice Teachers and their Instructors Give up too Easily? *Journal for Research on Mathematics Education, 23*(3), 194-222.

Brown, C., & Borko, H. (1992). Becoming a Mathematics Teacher. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 209-239). New York: Macmillan Publishing Co.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. (1999). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

CBMS, C. B. o. M. S. (2001a). *The Mathematical Education of Teachers Part I* . Washington, DC: Mathematical Association of America in cooperation with the American Mathematical Society.

CBMS, C. B. o. M. S. (2001b). *The Mathematical Education of Teachers Volume II*. Providence, RI: American Mathematical Society in cooperation with the Mathematical Association of America.

Education, N. C. f. R. o. T. (1988). *Teacher Education and Learning to Teach: A Research Agenda* .

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual Knowledge Falls through the Cracks: Complexities of Learning to Teach Mathematics for Understanding. *Journal for Research in Mathematics Education, 24*(1), 8-40.

Ely, M., Anzul, M., Friedman, T., Garner, D., & Steinmetz, A. M. (1991). *Doing Qualitative Research: Circles With Circles*. Pennsylvania: The Falmer Press, Taylor and Francis, Inc.

Feiman-Nemser, S., & Buchmann, M. (1985). *The First Year of Teacher Preparation: Transition to Pedagogical Thinking?* (Research Series No. 156). East Lansing, Michigan: The Institute for Reasearach on Teaching: Michigan State University.

Feiman-Nemser, S., & Buchmann, M. (1986). *When is Student Teaching Teacher Education?* (Reserach Series No. 178). East Lansing, Michigan: Institute for Research on Teaching.

Feiman-Nemser, S., & Remillard, J. (1995). *Perspectives on Learning to Teach* (Issue Paper 95-3). East Lansing, Michigan: National Center for Research on Teacher Learning.

Fennema, E., Carpenter, T. P., & Lamon, S. J. (Eds.). (1991). *Integrating Research on Teaching and Learning Mathematics*. Albany, NY: State University of New York Press.

Fennema, E., & Franke, M. L. (1992). Teachers' Knowledge and its Impact. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 147- 164). New York: Macmillan Publishing Co.

Grouws, D. A., & Cebulla, K. J. (2000). Elementary and Middle School Mathematics at the Crossroads. In T. L. Good (Ed.), *American Education: Yesterday, Today, and Tomorrow: Ninety-ninth Yearbook of the National Society for the Study of Education* (pp. 209-255). Chicago, Illinois: National Society for the Study of Education - NSSE.

Hiebert, J. (1999). Relationships Between Research and the NCTM Standards. *Journal for Research in Mathematics Education, 30*(1), 3-19.

Hiebert, J., & Carpenter, T. (1992). Learning and Teaching with Understanding. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 65-97). New York: Macmillan Publishing Co.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Oliver, A., & Human, P. (1997). *Making Sense: Teaching and Learning Mathematics with Understanding*. Portsmouth, NH: Heinemann.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahway, New Jersey: Lawrence Erlbaum Associates.

Manouchehri, A. (1997). School Mathematics Reform: Implications for Mathematics Teacher Preparation. *Journal of Teacher Education, 48*(3), 197, 115 p.

Mewborn, D. (2000, April 28, 2000). *An Analysis of the Research on K-8 Teachers' Mathematical Knowledge*. Paper presented at the American Educational Research Association, New Orleans.

NCRTL, N. C. f. R. o. T. L. (1991). *Findings from the Teacher Education and Learning to Teach Study*. East Lansing, Michigan: National Center for Research on Teacher Learning.

NCTM, N. C. o. T. o. M.-Ì. . (1989). *Curriculum and Evaluation Standards for School Mathematics*. Washington, D.C.: National Council of Teachers of Mathematics.

Patton, M. (1980). *Qualitative Evaluation*. Newbury Park, CA: Sage.

Peterson, P., Fennema, E., & Carpenter, T. (1991). Teachers' Knowledge of Students' Mathematics Problem-Solving Knowledge. In J. Brophy (Ed.), *Advances in Research on Teaching: Volume 2: Teacher's Knowledge of Subject Matter as it Relates to their Teaching Practice* (Vol. 2, pp. 49-85). Greenwich, Connecticut: JAI Press, Inc.

Putnam, R. T., & Borko, H. (2000). What do New Views of Knowledge and Thinking Have to Say About Research on Teacher Learning? *Educational Researcher, 29*(1), 4-15.

Schifter, D. (Ed.). (1996a). *What's Happening in Math Class: Envisioning New Practices Through Teacher Narratives*. New York: Teacher's College Press.

Schifter, D. (Ed.). (1996b). *What's Happening in Math Class: Reconstructing Professional Identities*. (Vol. 1). New York: Teacher's College Press.

Schifter, D., & Bastable, V. (1995). From the Teachers' Seminar to the Classroom: The Relationship Between Doing and Teaching Mathematics, An Example From Fractions.

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher* (February 1986), 4-14.

Simon, M. A. (1993). Prospective Elementary Teachers' Knowledge of Division. *Journal for Research in Mathematics Education, 24*(3), 233-254.

Sowder, J. T., Philipp, R. A., Armstrong, B. E., & Schappelle, B. P. (1998). *Middle-Grade Teachers' Mathematical Knowledge and Its Relationship to Instruction: A Research Monograph*. New York: State University of New York.

Stigler, J. W., Gallimore, R., & Hiebert, J. (2000). Using Video Surveys to Compare Classrooms and Teaching across Cultures: Examples and Lessons from the TIMSS Video Studies. *Educational Psychologist, 35*(2), p. 87, 14 p.

Yaffee, L. (1996). Pictures at an Exhibition: A Mathphobic Confronts Fear, Loathing, Cosmic Dread, and Thirty Years of Math Education. In D. Schifter (Ed.), *What's Happening in Math Class: Reconstructing Professional Identities: Volume 2*. New York: Teachers College Press.

Appendix: Summary of Teaching Experiences

Name	Student Teaching	Teaching Positions Prior to Study	Position in 2000-1 (9/00-6/01 unless stated otherwise)	Position in 2001-2
Beth	Spring, 1998 Sixth grade	Teacher 4/99-6/00 7th grade math in public Urban 6-8 public school	7th grade math in middle school	Same
George	Spring, 1996 Third/fourth	9/96-6/01, Teacher's assistant, grades 1/2 Urban K-8 public school	Teacher- Grade 1/2 Urban private elementary school	Same
Helen	Spring, 1996 Grades 5/6	9/96-97 Teacher's aide	1997 on Teacher, grade 5 in middle school, grades 5-8, suburban public	Same
Jan	Fall, 1998 Grades 1/2	9/99-6/00 Teacher, grade 2 Urban public school	9/00-6/01 Teacher, grade 2, Urban public school	Reading, Urban public school
Rita	Spring, 1999 Grades 1/2	9/99-6/00 Associate Teacher, Kindergarten Private Urban School 9/00-6/01 Team	Teacher, Kindergarten, Private urban school.	Teacher, kindergarten, suburban public school
Sara	Spring, 1997 Grades 3/4	1/98- 6/98 Teacher, 5th grade, public suburban 9/98-12/99 Aide 4th grade suburban public 1/00-6/00 Teacher 4th grade suburban public 9/00-12/00 Aide, 4th grade suburban	12/00-6/01, Teacher, 3rd grade suburban public	Teacher, 2nd grade, suburban public

Tina	Spring, 1998 Grades 1/2	98-99 Teacher's Aide, 1/2 public urban 99-00 pre-K parochial 9/00-12/00 Aide	12/00-6/01 Third grade reading and math urban, public	Teacher, first grade, suburban public